Is electric polarization well-defined in a Chern insulator?

Sinisa Coh and David Vanderbilt

Department of Physics & Astronomy, Rutgers University
Piscataway, NJ 08854-8019, USA

In 1988, Haldane pointed out the possibility that an insulating crystal with broken time-reversal symmetry might exhibit a quantized Hall conductance even in the absence of a macroscopic magnetic field.[1] We shall refer to such a material as a "Chern insulator" because it necessarily would have a non-zero Chern invariant associated with its manifold of occupied Bloch states.

Because a Wannier representation does not exist for a Chern insulator,[2] this cannot provide a path to a definition of polarization. We show that there is also a fundamental difficulty with the naive use of the Berry-phase definition, namely, that it would yield a polarization that depends on the choice of the boundaries of the Brillouin zone. We analyze why such an approach fails, and from there we prescribe the proper way to interpret polarization obtained via the Berry-phase definition. Such an interpretation reduces to the usual electric polarization in the case of an ordinary insulator, and is related to observables such as the internal current that flows in response to a change of the underlying crystal Hamiltonian, or to the surface charge at the surface of the crystal. The connection to surface charges is especially subtle because of the partial filling of chiral metallic edge states that are necessary associated with a Chern insulator. We also compare the polarization for normal and Chern insulators, and clarify the fundamental differences between the two.

In order to test our findings, we calculated the polarization of large but finite 2D samples cut from the bulk Haldane model. For such finite systems the polarization can be straightforwardly calculated and we show that it is compatible up to numerical accuracy with our interpretation of the Berry-phase polarization.

[1] F. D. M. Haldane, Phys. Rev. Lett. **61**, 2015 (1988),

[2] T. Thonhauser and D. Vanderbilt, Phys. Rev. B **74**, 235111 (2006).