

# Ab initio calculations of electronic response: ingredients, results and challenges



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<http://etsf.polytechnique.fr>

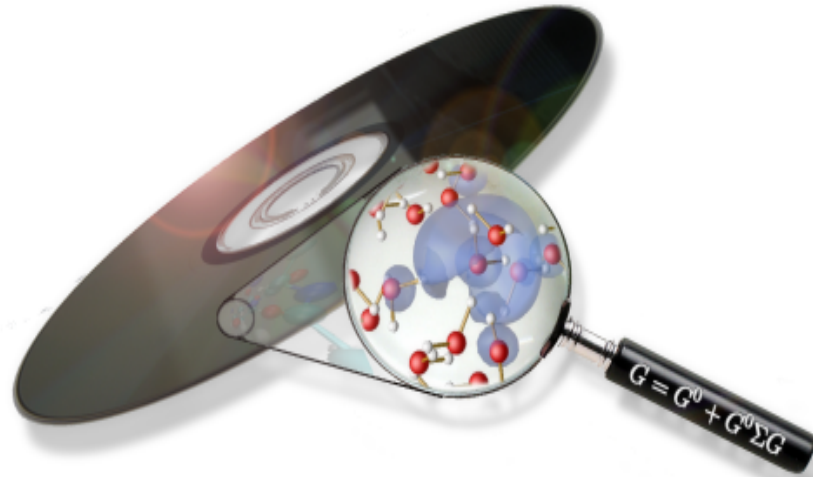
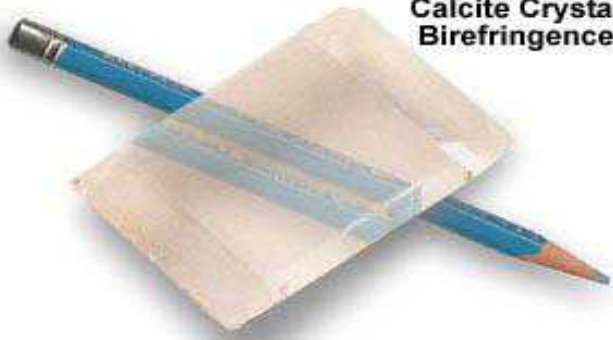


# Electronic response.....

- 1) .....importance and challenge
- 2) **Induced potentials: Hartree**
  - \* **Example: plasmon dispersion in graphene, tubes**
- 3) **Induced potentials: xc**
  - \* **Example: IXS for silicon**
- 4) **Honey or balls? GW+BSE**
- 5) **Frontiers I: localisation, correlation**
  - \***Examples: TMO, CIS**
- 6) **Frontiers II: merge**
- 7) **Frontiers III: compromise**
- 8) **Summary**

# Electronic response.....

Calcite Crystal  
Birefringence

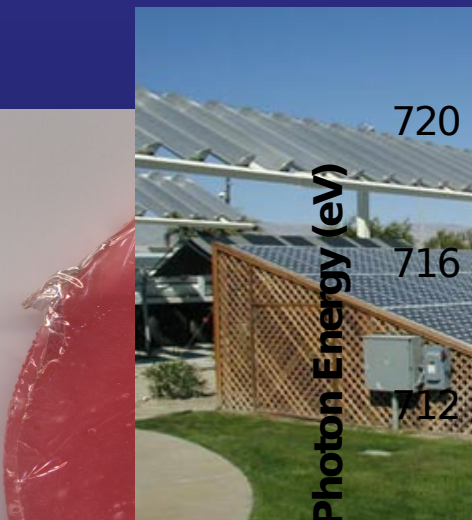
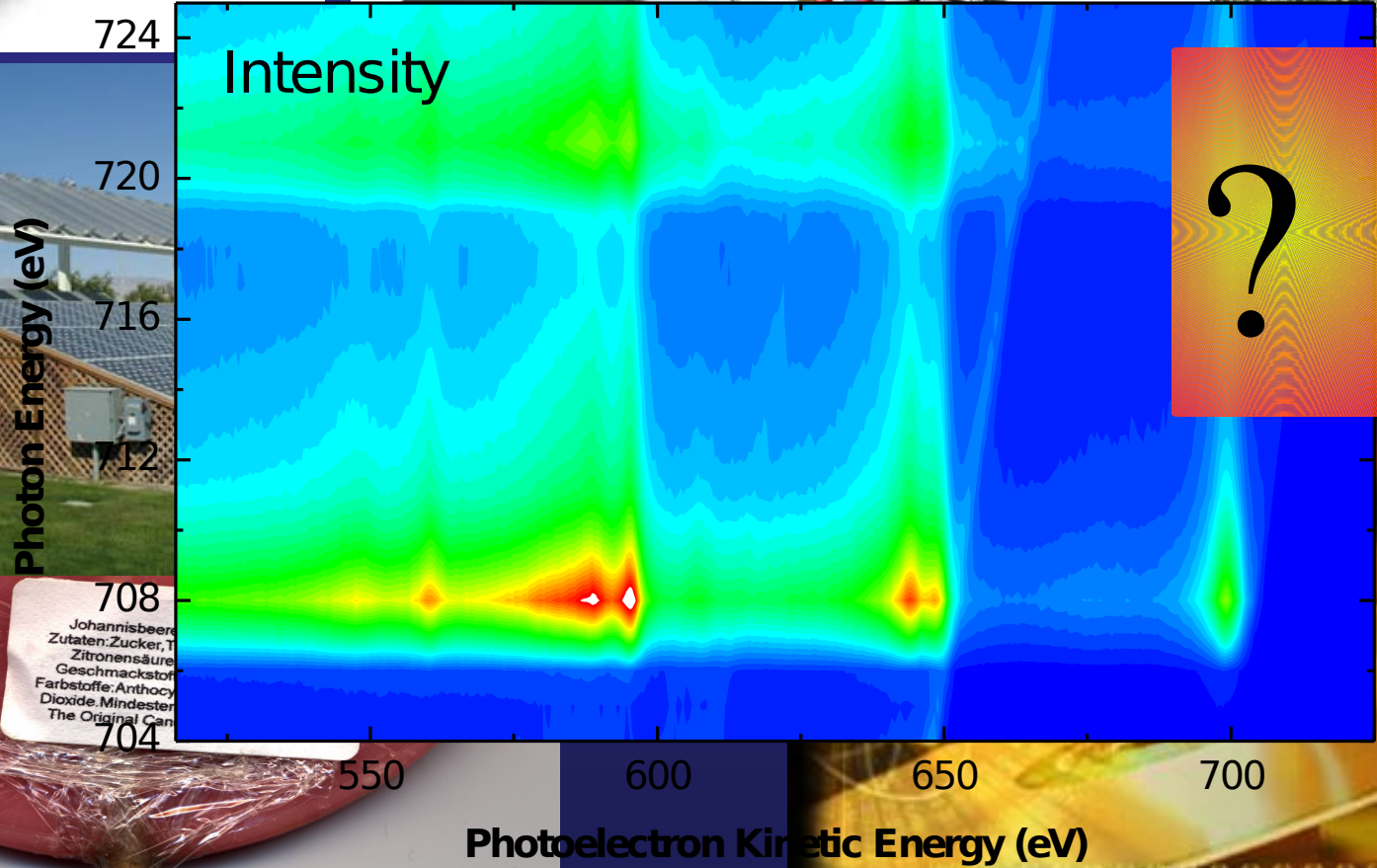
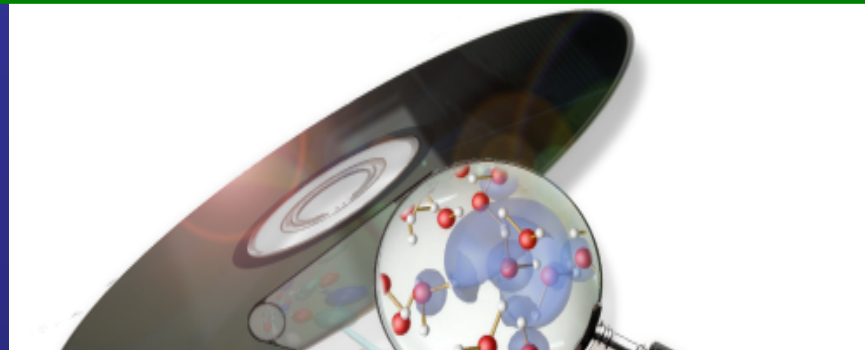
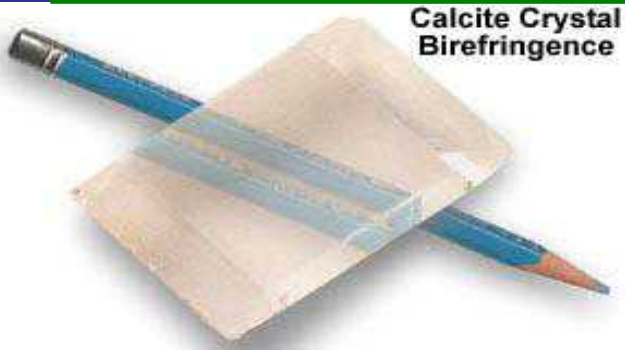


Johannisbeere Lucky Pop  
Zutaten: Zucker, Traubenzucker,  
Zitronensäure, Natürliche  
Geschmacksstoffe, Natürliche  
Farbstoffe: Anthocyanins, Titanium  
Dioxide. Mindestens Haltbar Bis:  
The Original Candy Company

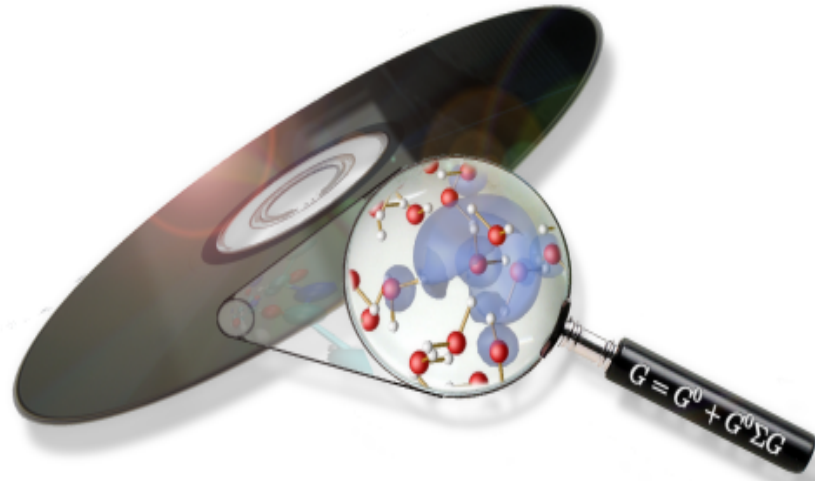
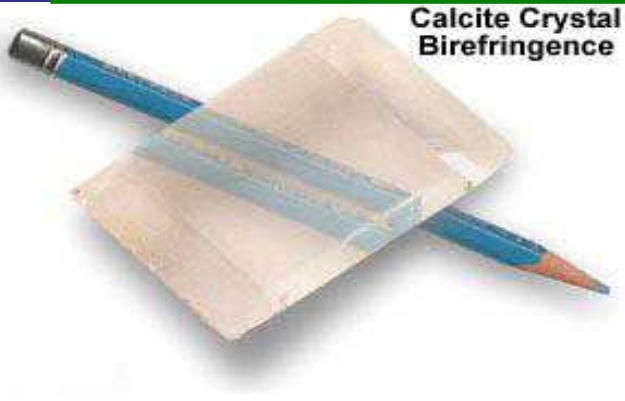
10/20/05



# Electronic response.....



# Electronic response.....



$$H\phi(x_1, \dots, x_N) = E \phi(x_1, \dots, x_N)$$



Johannisbeere Lucky Pop  
Zutaten: Zucker, Traubenzucker,  
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The Original Candy Company



# Electronic response.....

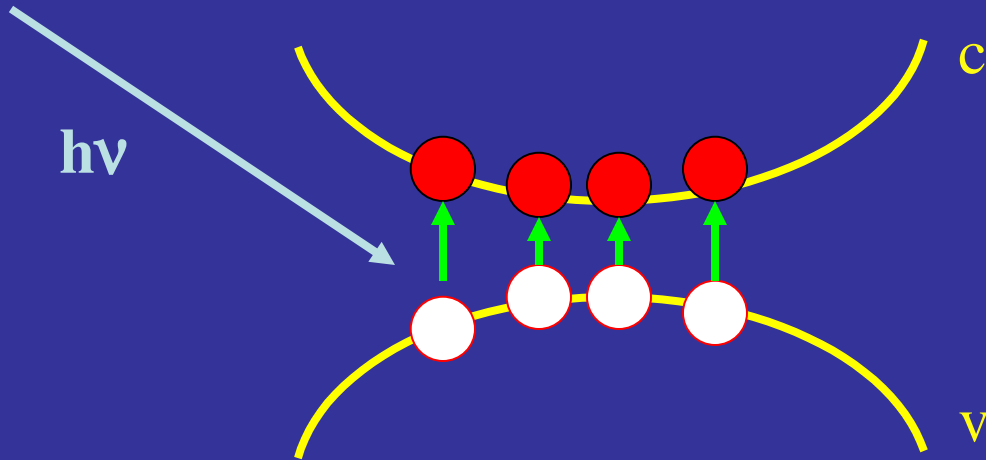
realistic

feasible

**Calculations**

easy to interpret

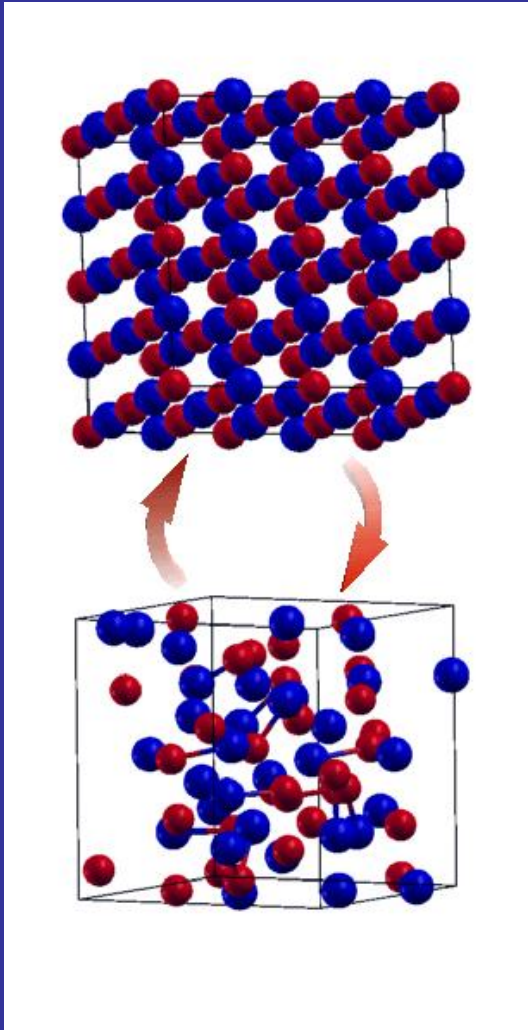
# Independent electrons and transitions



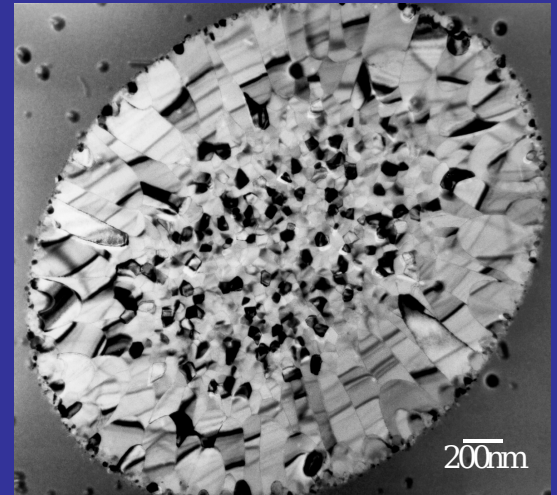
$$\text{Im} [\chi_0] \sim \sum_{vc} |\langle v|D|c\rangle|^2 \delta (E_c - E_v - \omega)$$

Energies and wavefunctions often from DFT – Kohn Sham

# PCM: Reversible phase transition



GeSbTe alloys:

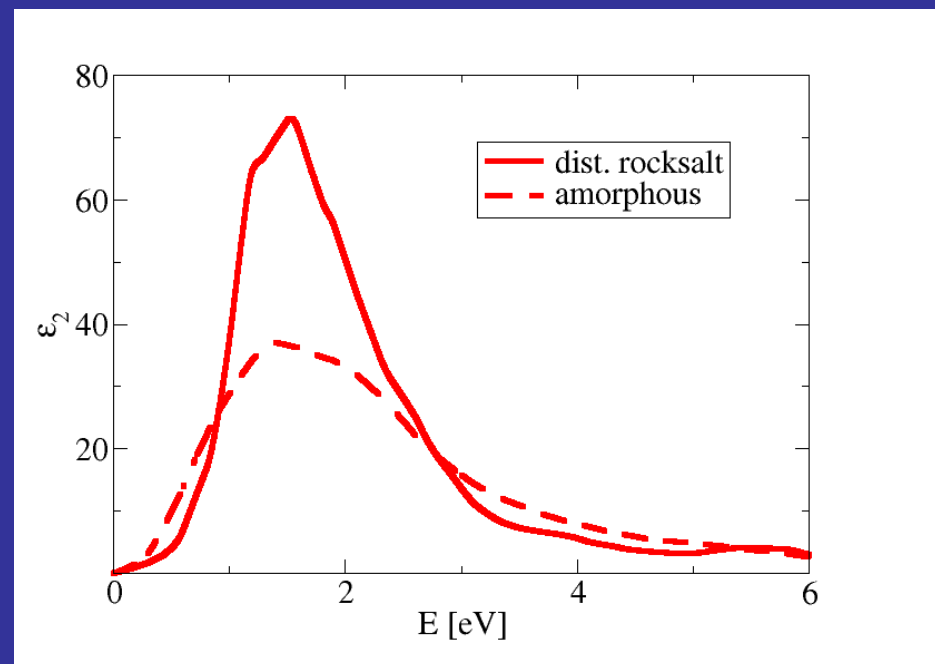
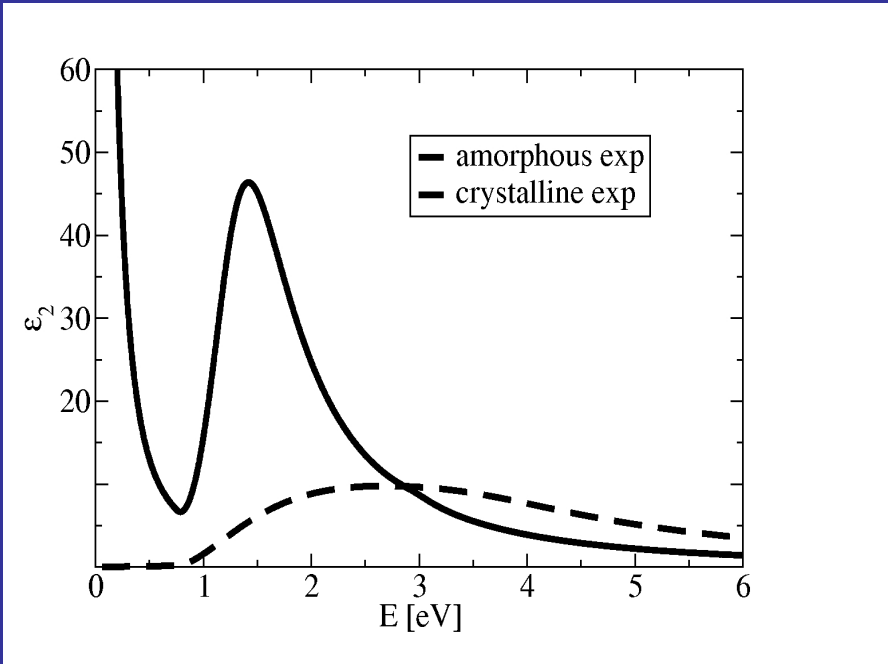


- rapid transition:  $\sim 10-100\text{ns}$
- large optical & electric contrast
- although already applied still not well understood



# calculations & experiment

- spectroscopy on polycrystalline thin films of GeTe
- optical spectra calculations (RPA)

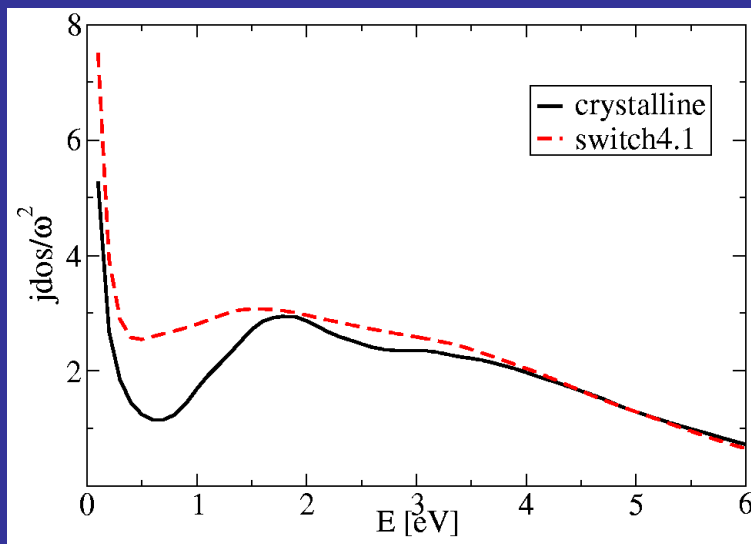


change in local order results in change of optical absorption.

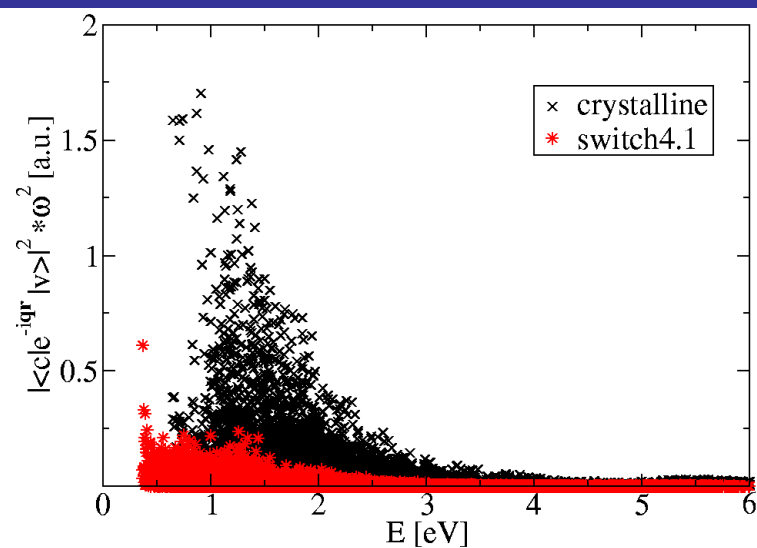
# Origin of strong change in absorption

Absorption given by Fermi's golden rule:

$$\text{Im}(\epsilon_M(\omega)) = 2\pi \lim_{\mathbf{q} \rightarrow 0} v_0(\mathbf{q}) \sum_{vc} |\langle c | e^{-i\mathbf{q}\cdot\mathbf{r}} | v \rangle|^2 \delta(\omega - (\epsilon_c - \epsilon_v))$$



No significant changes in jdof

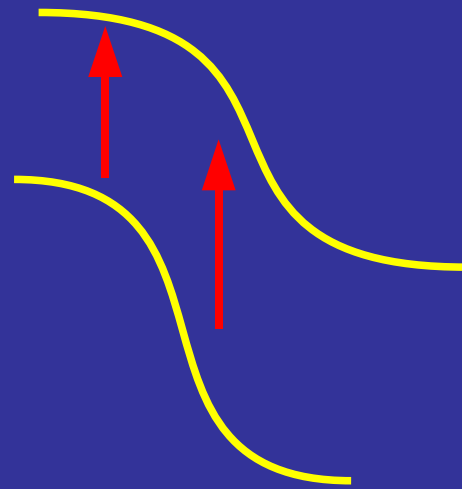


Matrix elements define  
optical contrast!

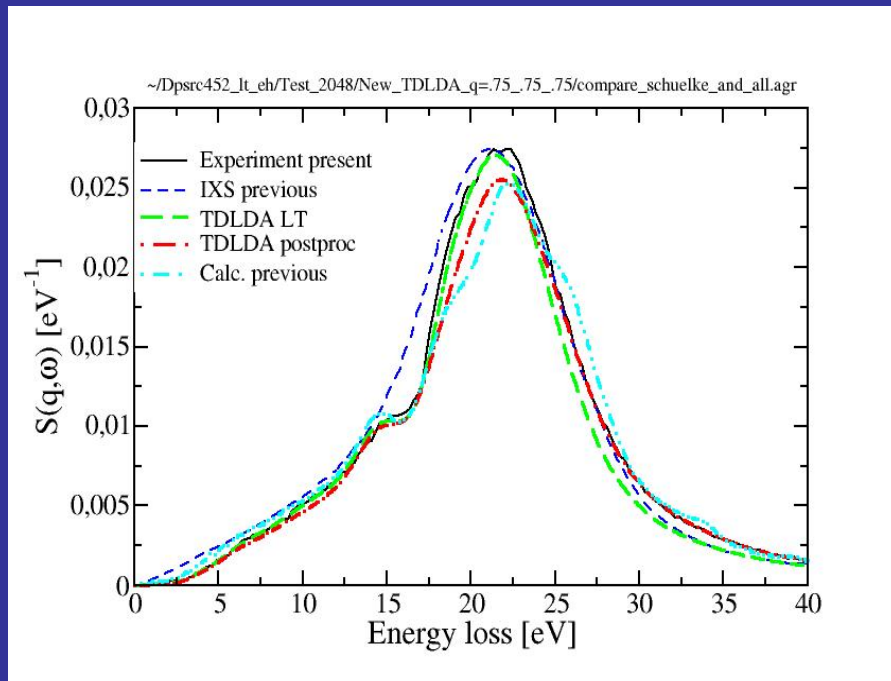
# **PFlop:**

- from model short range amorphous to realistic material
- defects
- many body effects for localized states

# Silicon:



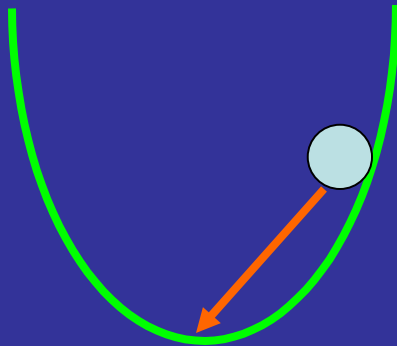
3 – 5 eV



20 eV

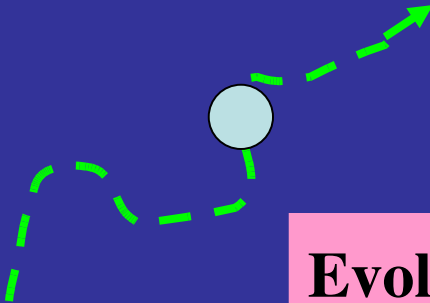
# Time-Dependent Density Functional Theory

Equilibrium position: minimization of  $E$



Ground state: DFT  $\min[E]$

Density propagation

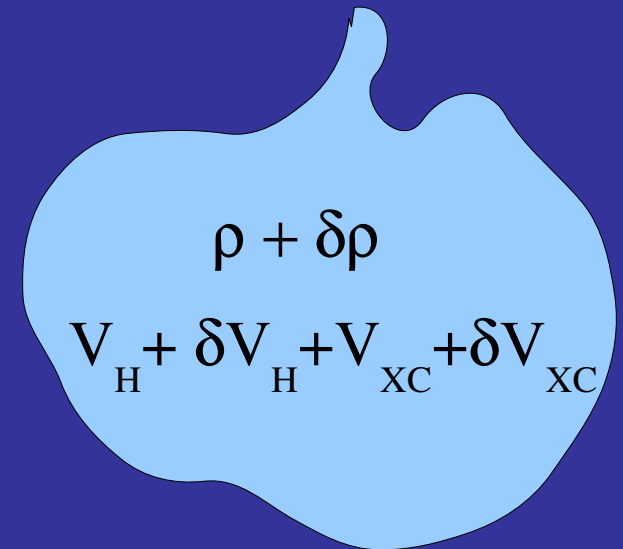
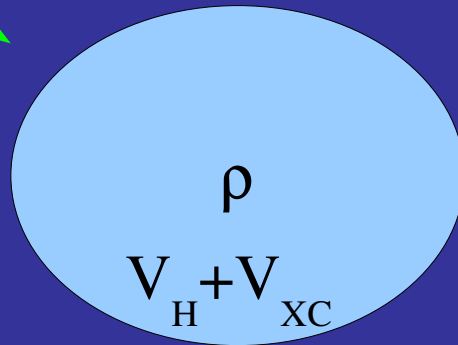


Trajectory: extrema of the action  $A$

**Evolution of the system (its density) due to external field: TD-DFT  $[A]$**

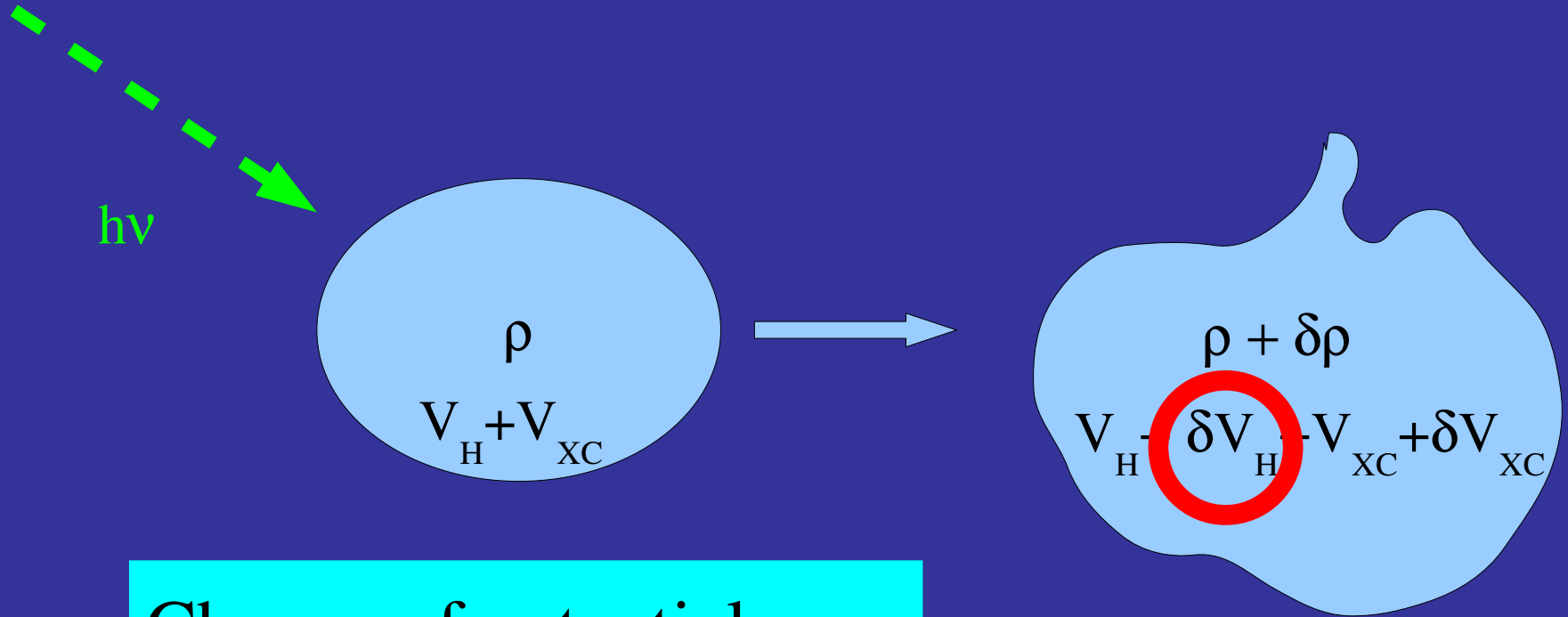
Excitation ?

$h\nu$



Change of potentials

Excitation ?



Change of potentials

Induced Hartree: long-range and local field effects

RPA

$$\delta V_{xc}/\delta\rho$$

TDDFT-RPA:  $\chi = \chi_0 + \chi_0 [ v ] \chi$

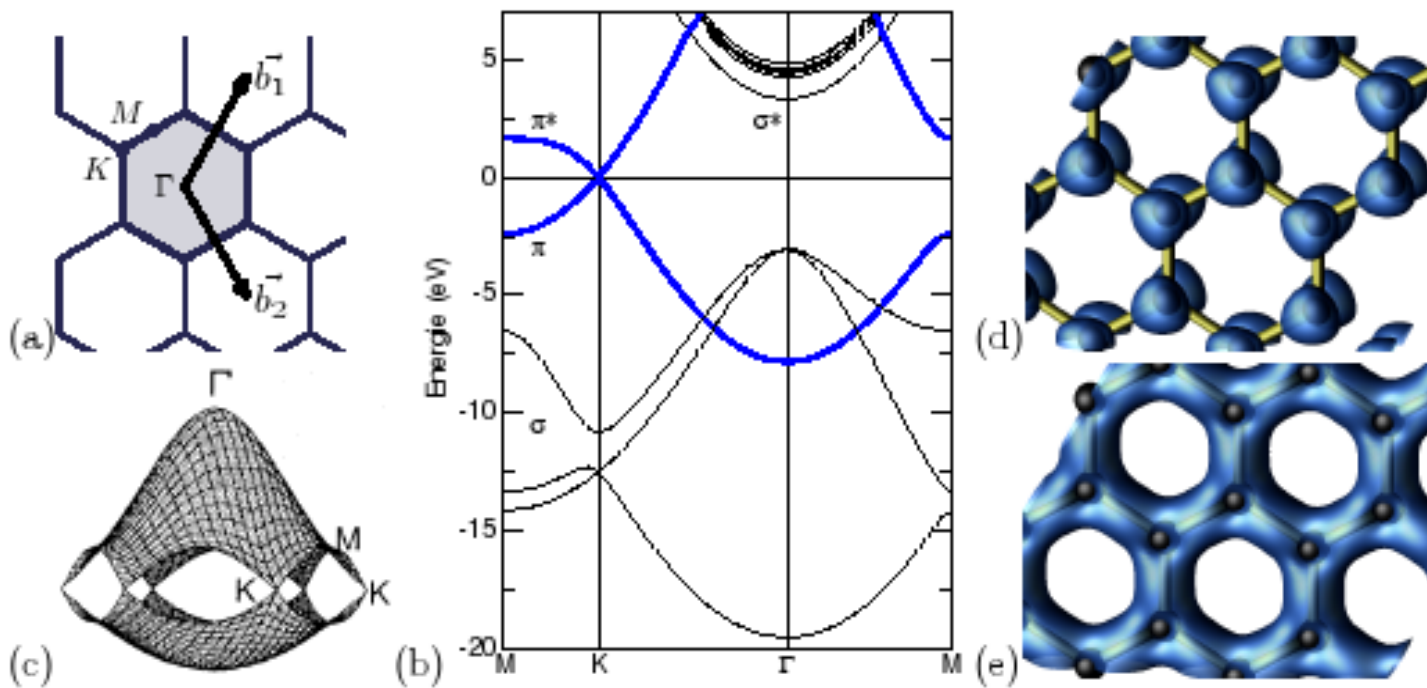
$$v(q+G) \sim 1/|q+G|^2$$

$G=0 \rightarrow$  plasmon

$G \neq 0 \rightarrow$  crystal local field effects

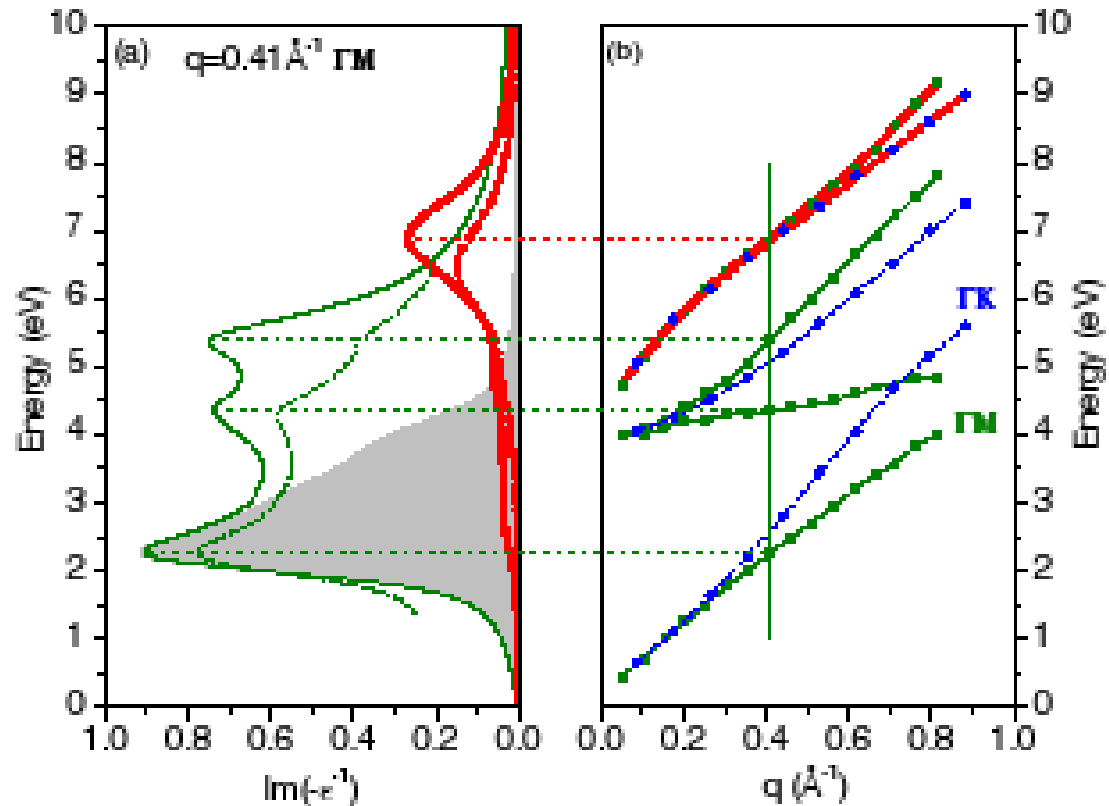


# Graphene, $\pi$ plasmon



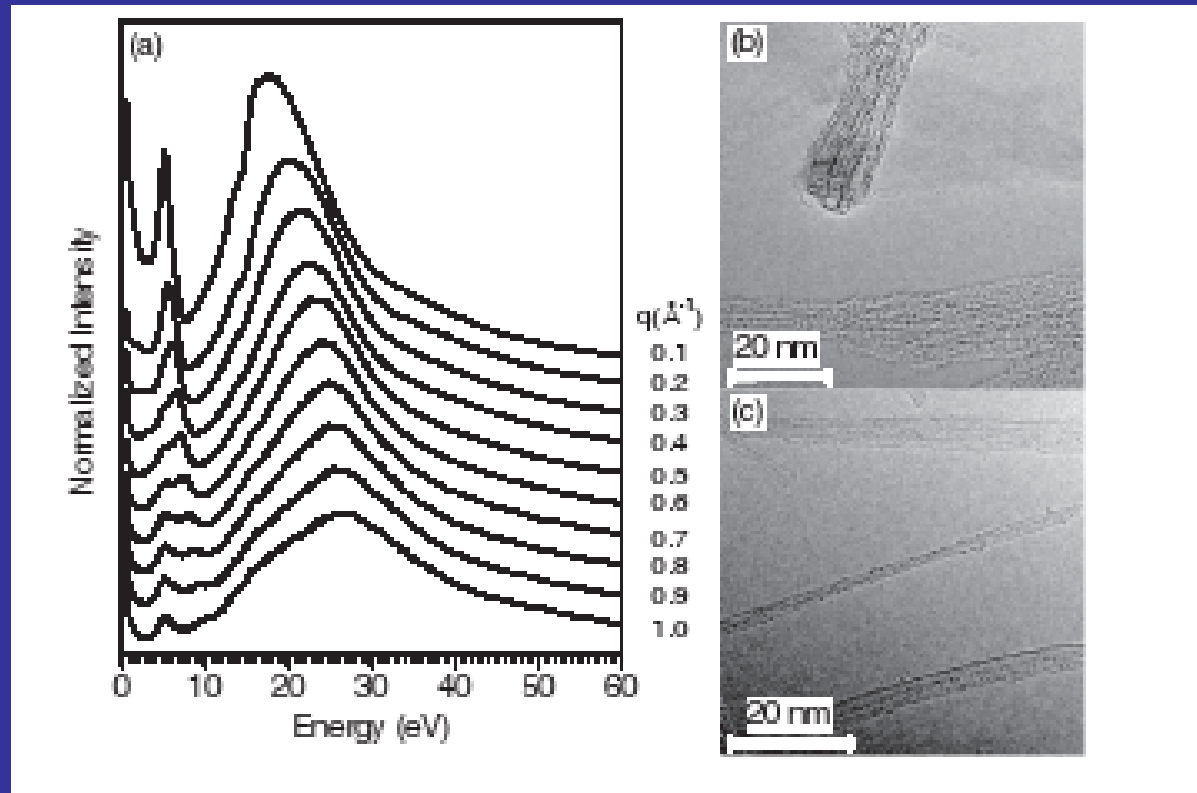
R. Hambach, Diplomarbeit

# Graphene, $\pi$ plasmon



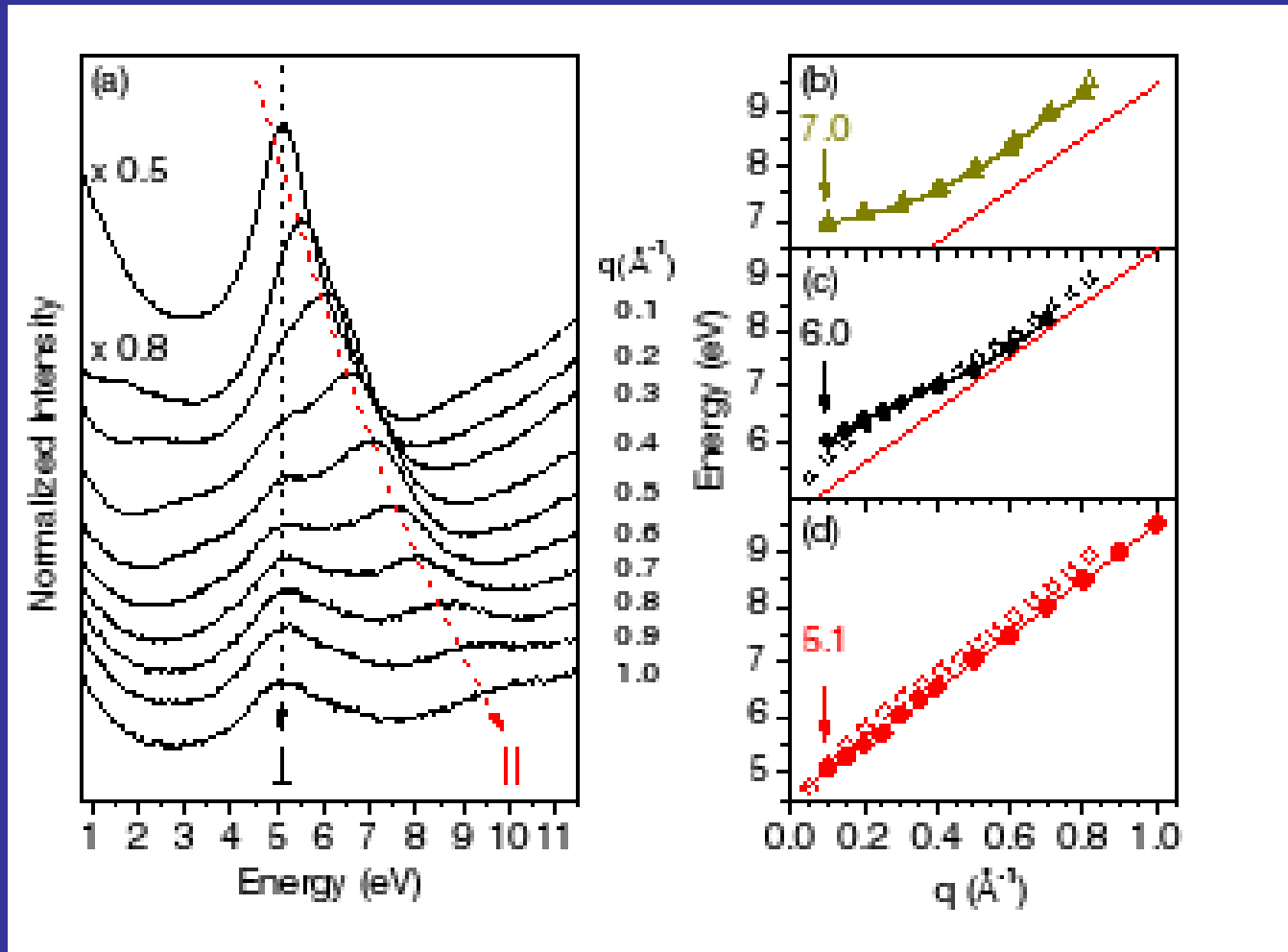
C. Kramberger, R. Hambach et al., PRL in press (2008)

# Isolated carbon nanotube, $\pi$ plasmon



C. Kramberger et al., PRL 100, 196803 (2008)

# Nanotubes and graphene, $\pi$ plasmon dispersion

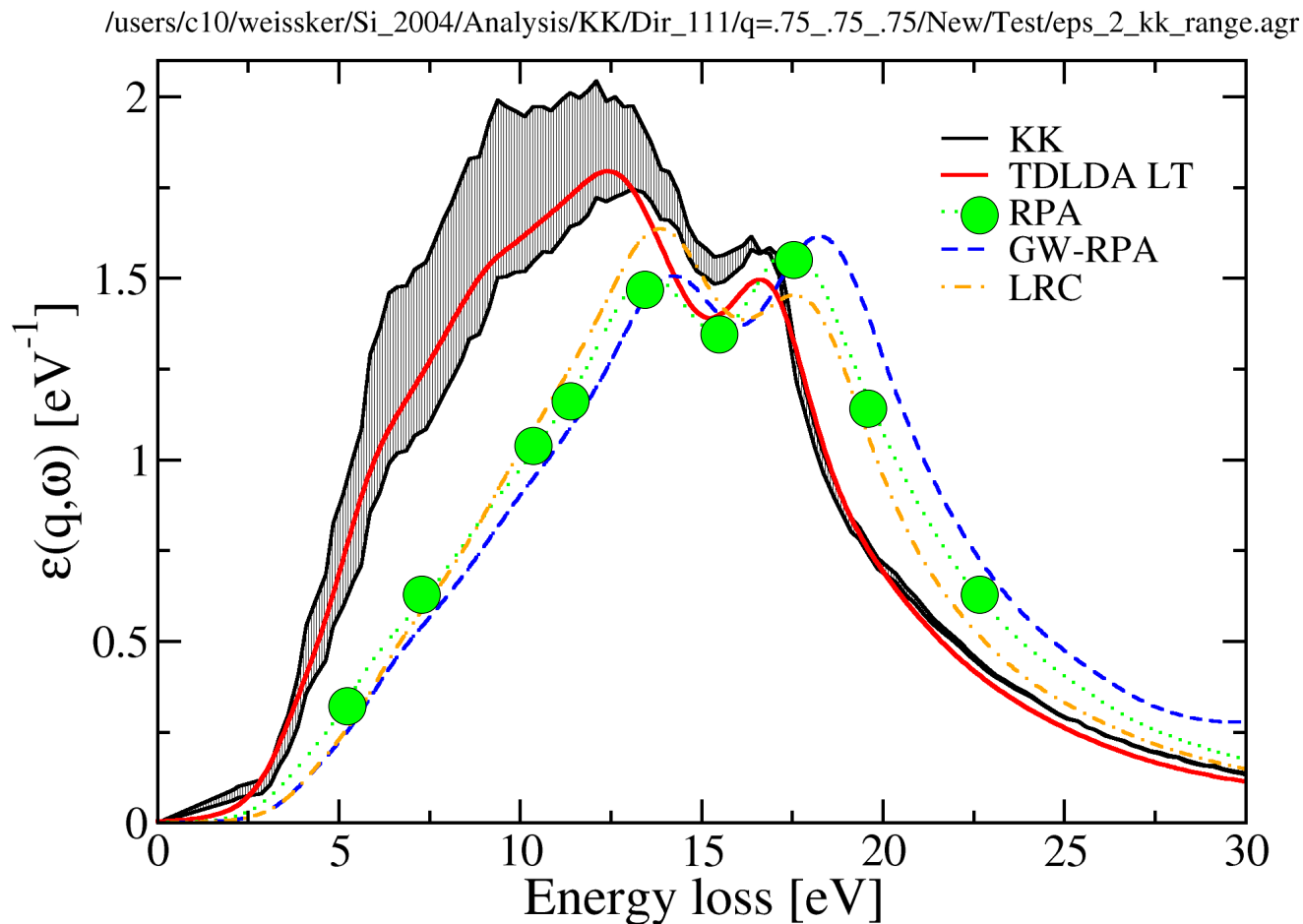


C. Kramberger et al., PRL 100, 196803 (2008)

# **PFlop:**

- larger nanotubes
- multiwall
- doping

# More quantitatively?



H. Weissker et al., Phys. Rev. Lett. **97**, 237602 (2006)  
Collaboration LSI – ESRF (ID16)

$$\delta V_{xc}/\delta\rho$$

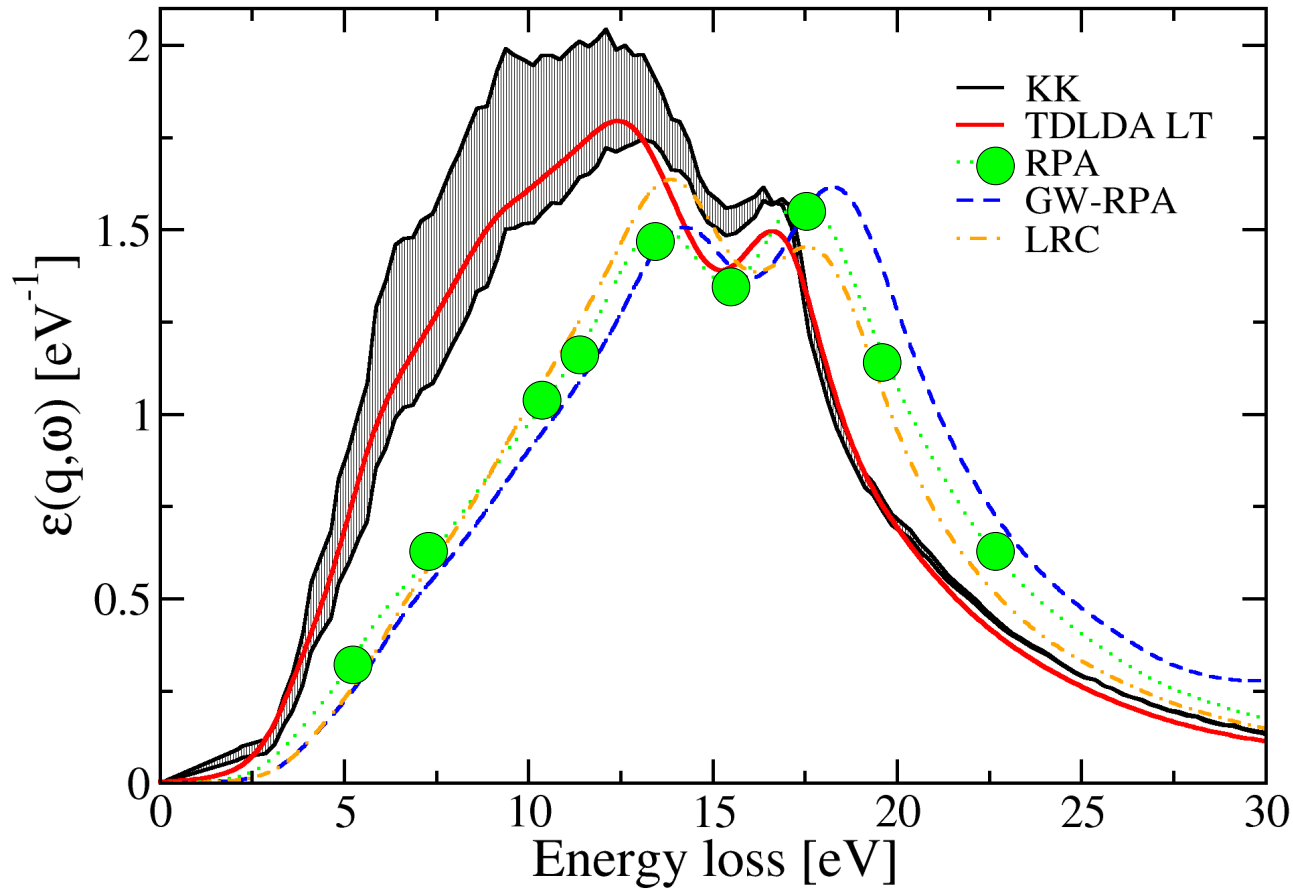
TDDFT

$$\chi = \chi_0 + \chi_0 [ v + f_{xc} ] \chi$$

$$\delta V_{xc}/\delta\rho$$

$$\text{TDLDA: } \delta V_{xc}^{\text{LDA}}(r,t)/\delta\rho(r',t') = \delta(r-r')\delta(t-t')dV_{xc}/d\rho$$

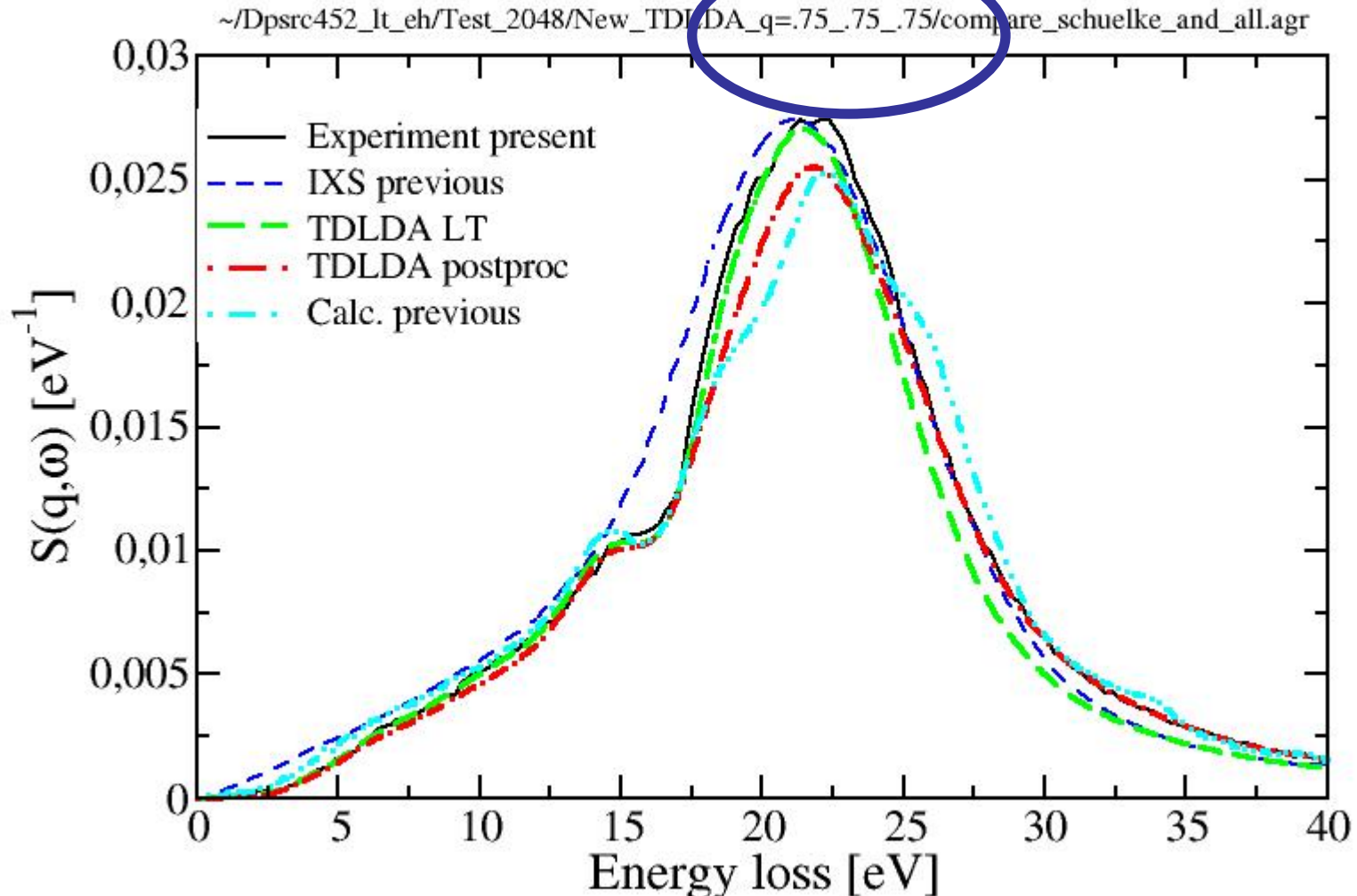
/users/c10/weissker/Si\_2004/Analysis/KK/Dir\_111/q=.75\_.75\_.75/New/Test/eps\_2\_kk\_range.agr



H. Weissker et al., Phys. Rev. Lett. **97**, 237602 (2006)  
Collaboration LSI – ESRF (ID16)

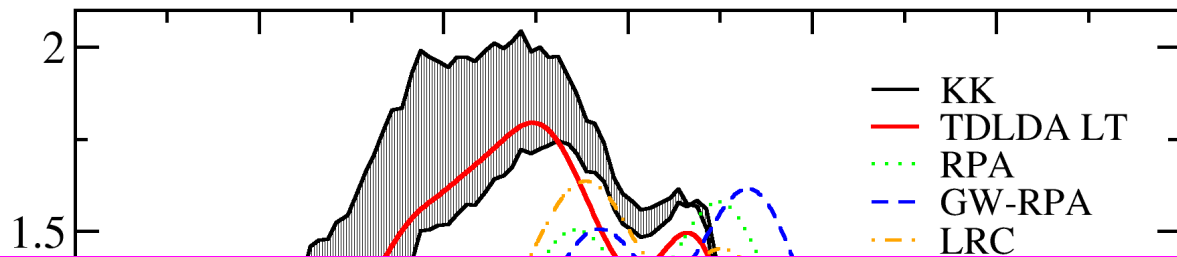


Fortunately.....TDLDA often works

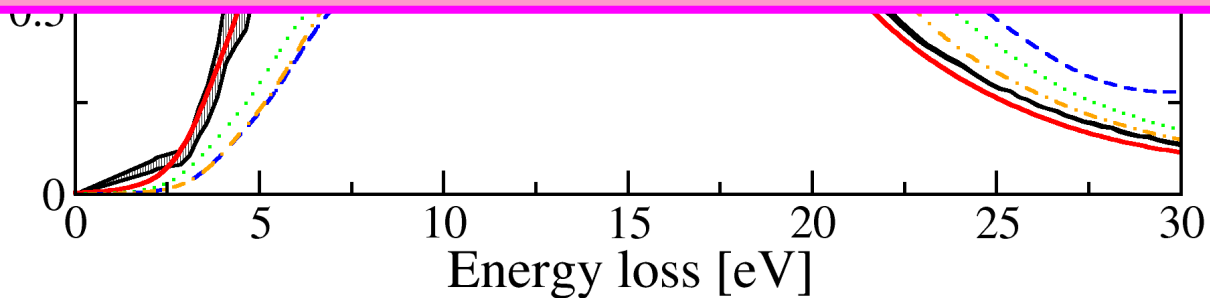


H. Weissker et al., Phys. Rev. Lett. **97**, 237602 (2006)  
Collaboration LSI – ESRF (ID16)

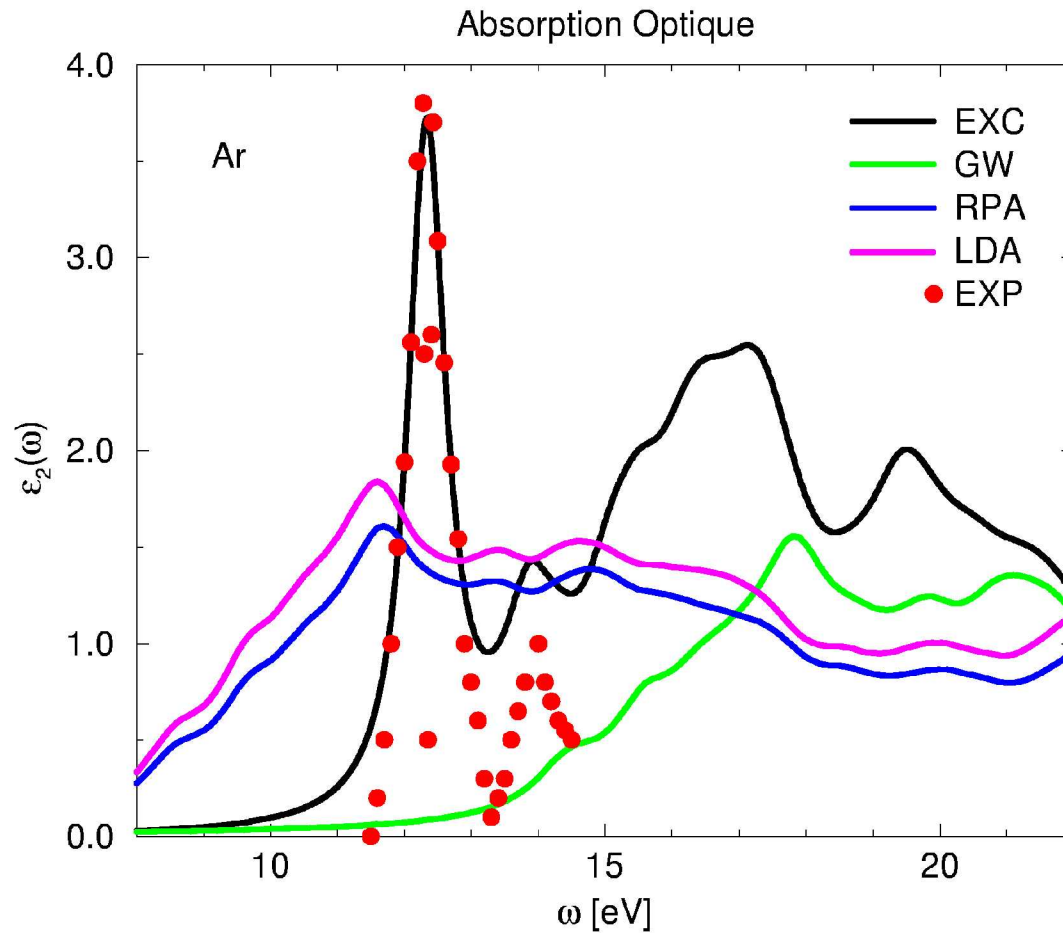
/users/c10/weissker/Si\_2004/Analysis/KK/Dir\_111/q=.75\_.75\_.75/New/Test/eps\_2\_kk\_range.agr



**TD-LDA yields excellent loss and  $\text{Im}(\epsilon)$  for finite momentum transfer !!!**



H. Weissker et al., Phys. Rev. Lett. **97**, 237602 (2006)  
Collaboration LSI – ESRF (ID16)



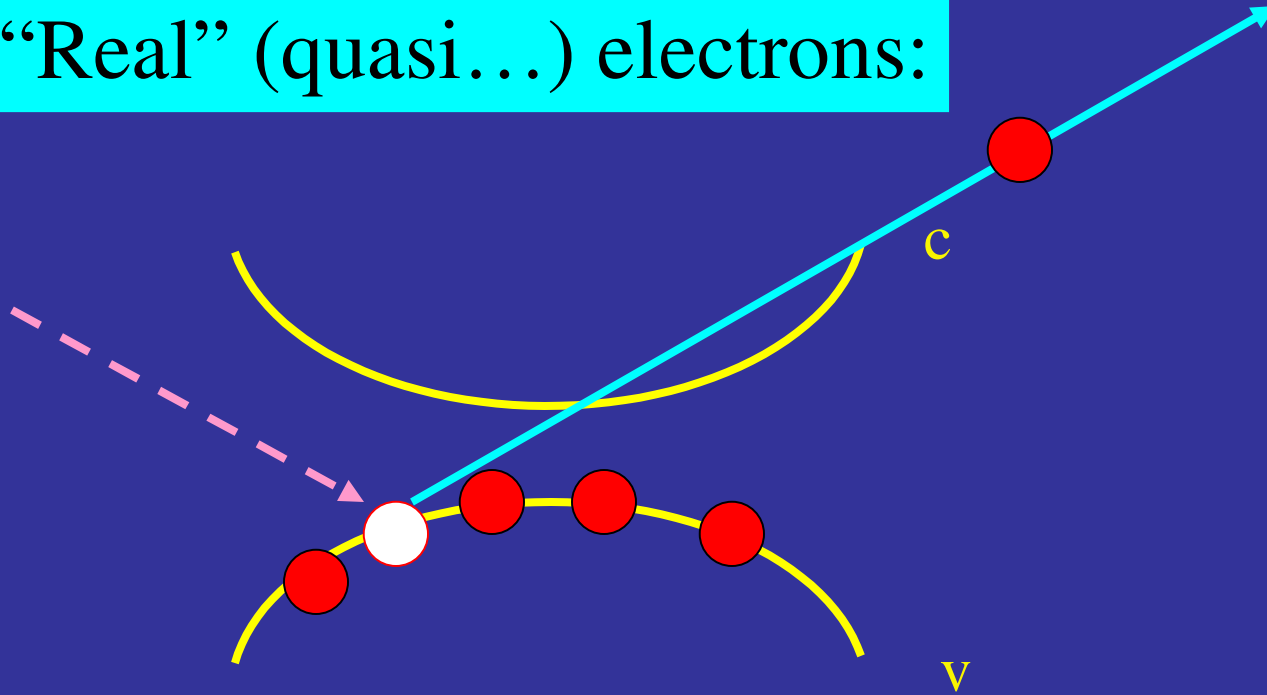
**RPA and TDLDA: sometimes quite wrong!**

$E_c - E_v$  (minimum, eV)

	DFT-KS	Exp.
Silicon	0.55	1.17
Diamond	4.26	5.48
MgO	5.3	7.83
Ar	8.2	14.2

**KS underestimates gaps  
(transition energies)**

“Real” (quasi...) electrons:



**Hole - (N-1) electrons**

## Exchange

## Relaxation - dynamical correlations

$$(-\nabla^2/2 + V_{\text{ext}} + V_{\text{H}} + V_{\text{xc}}(\mathbf{r})) |n\rangle = E_n |n\rangle$$



$$(-\nabla^2/2 + V_{\text{ext}} + V_{\text{H}} + \Sigma) |n\rangle = E_n |n\rangle$$

$V_{\text{xc}}(\mathbf{r})$  to  $\Sigma(\mathbf{r},\mathbf{r}',E_n)$

$$\mathbf{W} = \boldsymbol{\epsilon}^{-1} \mathbf{v} \quad !!!$$

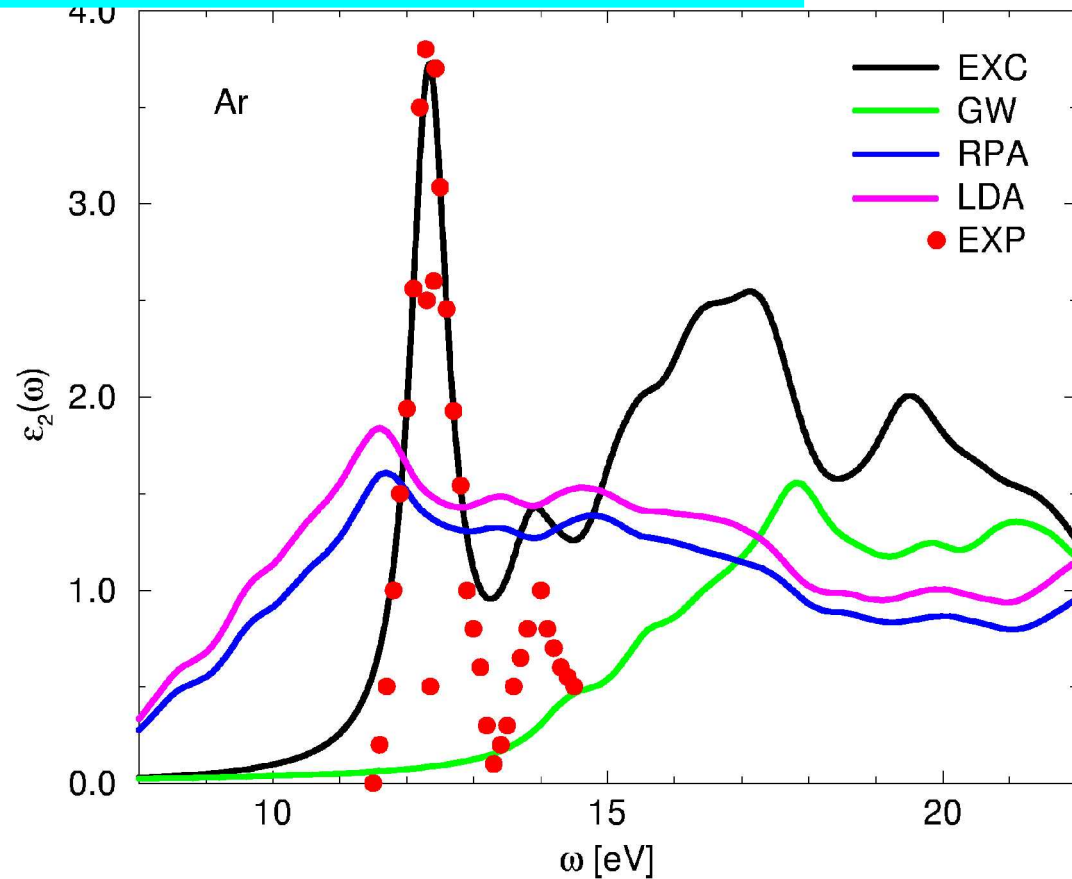
$$\Sigma = iGW \quad \text{L. Hedin, PR 1965}$$

$E_c - E_v$  (minimum, eV)

	DFT-KS	$\Sigma=iGW$	Exp.
Silicon	0.55	1.19	1.17
Diamond	4.26	5.64	5.48
MgO	5.3	7.8	7.83
Ar	8.2	14.0	14.2

**GW contains screening of hole  
(or electron)  $\rightarrow$  correct energies**

But back to absorption.....

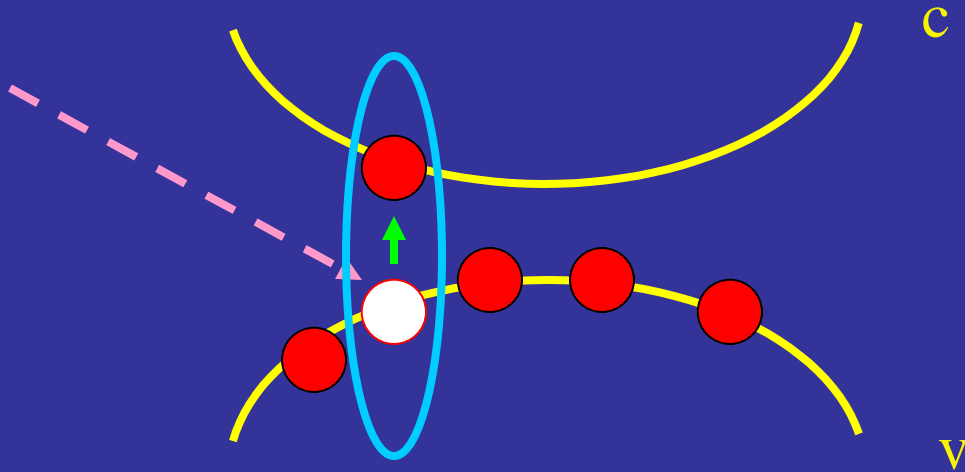


Even good bandstructure

→ sometimes quite wrong!



Absorption ?



Electron-hole interaction

Excitonic effects

**Bethe-Salpeter Equation**

# GW + Bethe Salpeter Equation

$$(H_{\text{el}} + H_{\text{hole}} + H_{\text{el-hole}}) A_{\lambda} = E_{\lambda} A_{\lambda}$$

$$\text{Im} [\epsilon] \sim \sum_{vc} |\langle v|D|c\rangle|^2 \delta(E_c - E_v - \omega)$$

$$\text{Im} [\epsilon] \sim \sum_{\lambda} \left| \sum_{vc} \langle v|D|c\rangle A_{\lambda}^{vc} \right|^2 \delta(E_{\lambda} - \omega)$$

->Mixing of transitions

->Modification of excitation energies

From  $\delta V_H / \delta \rho + \delta \Sigma / \delta G$

$$(H_{el} + H_{hole} + H_{el-hole}) A_\lambda = E_\lambda A_\lambda$$

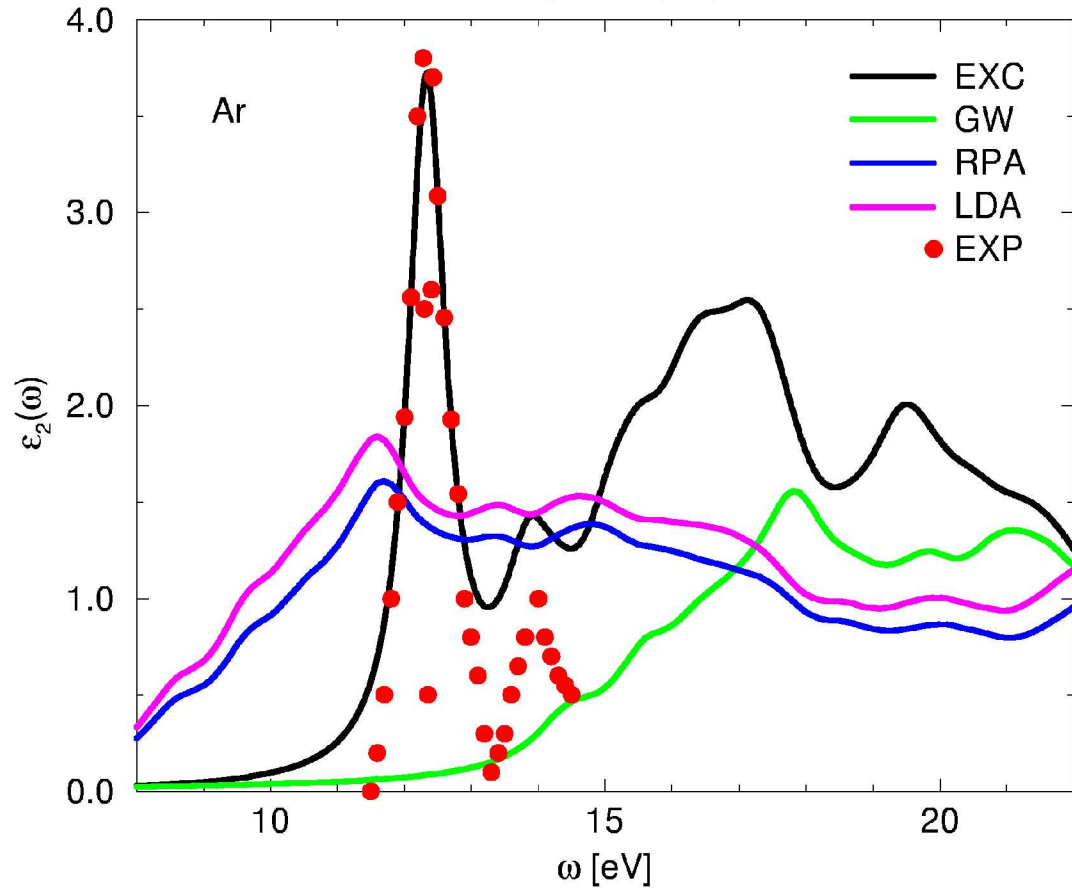
$$\text{Im} [\epsilon] \sim \sum_{vc} |\langle v | D | c \rangle|^2 \delta (E_c - E_v - \omega)$$

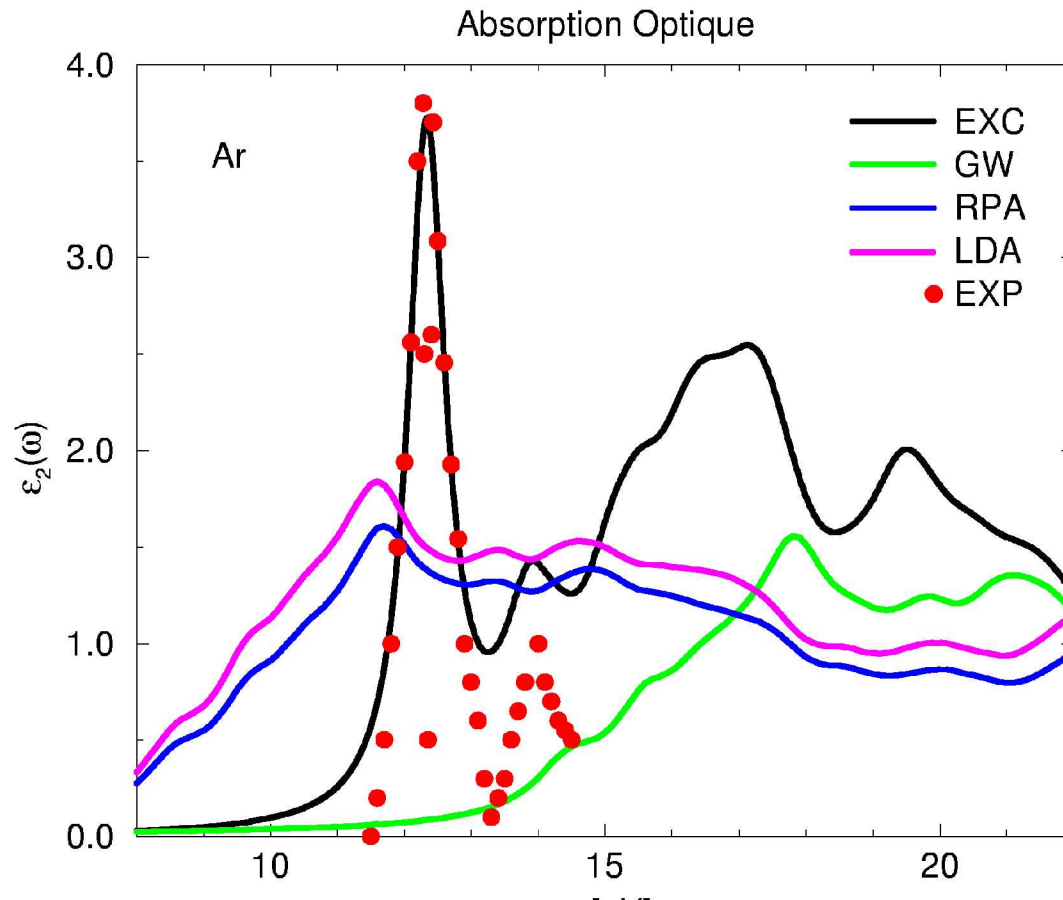
$$\text{Im} [\epsilon] \sim \sum_\lambda \left| \sum_{vc} \langle v | D | c \rangle A_\lambda^{vc} \right|^2 \delta (E_\lambda - \omega)$$

-> Mixing of transitions

-> Modification of excitation energies

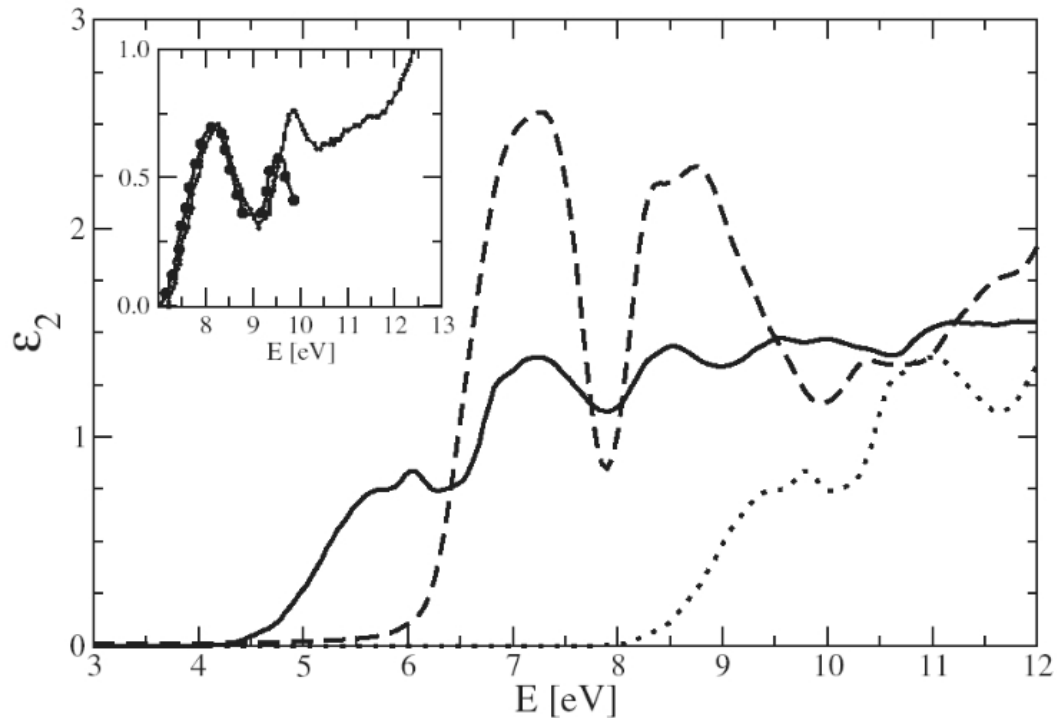
# Absorption Optique





**Electron-hole interaction  
for good absorption spectra**

# Excitons in water

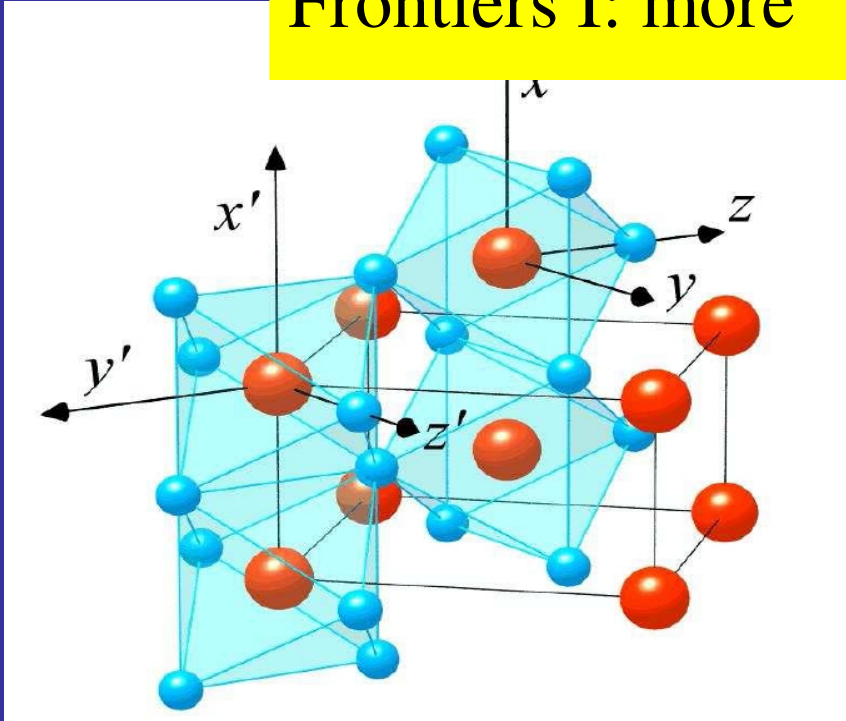


V. Garbuio, et al., PRL. **97**, 137402 (2006)

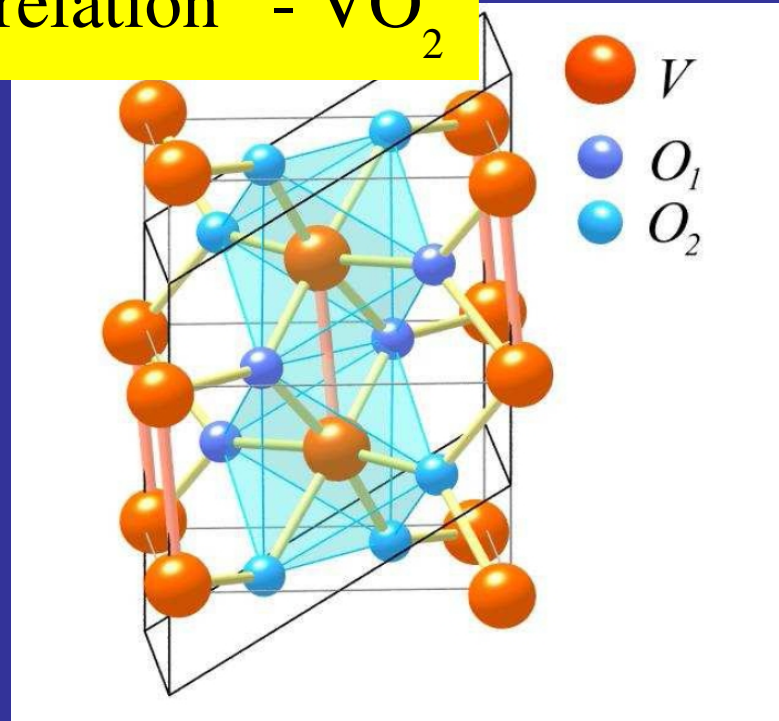
# **PFlop:**

- from model liquid to different phases of water
- molecules in solution

Frontiers I: more “correlation” -  $\text{VO}_2$

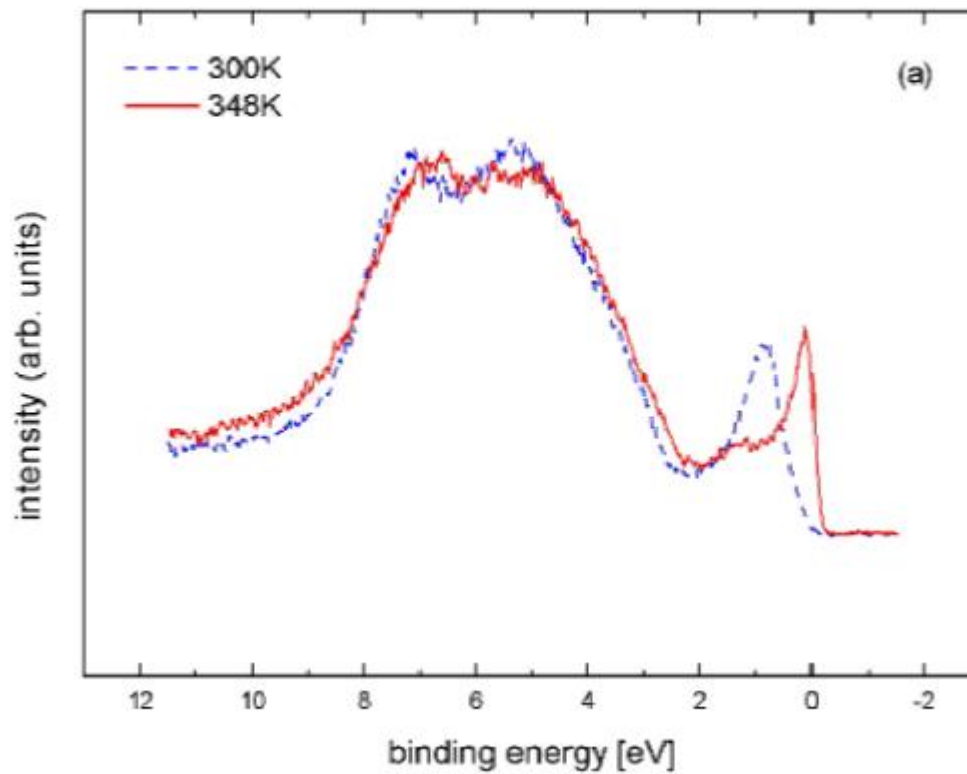


rutile

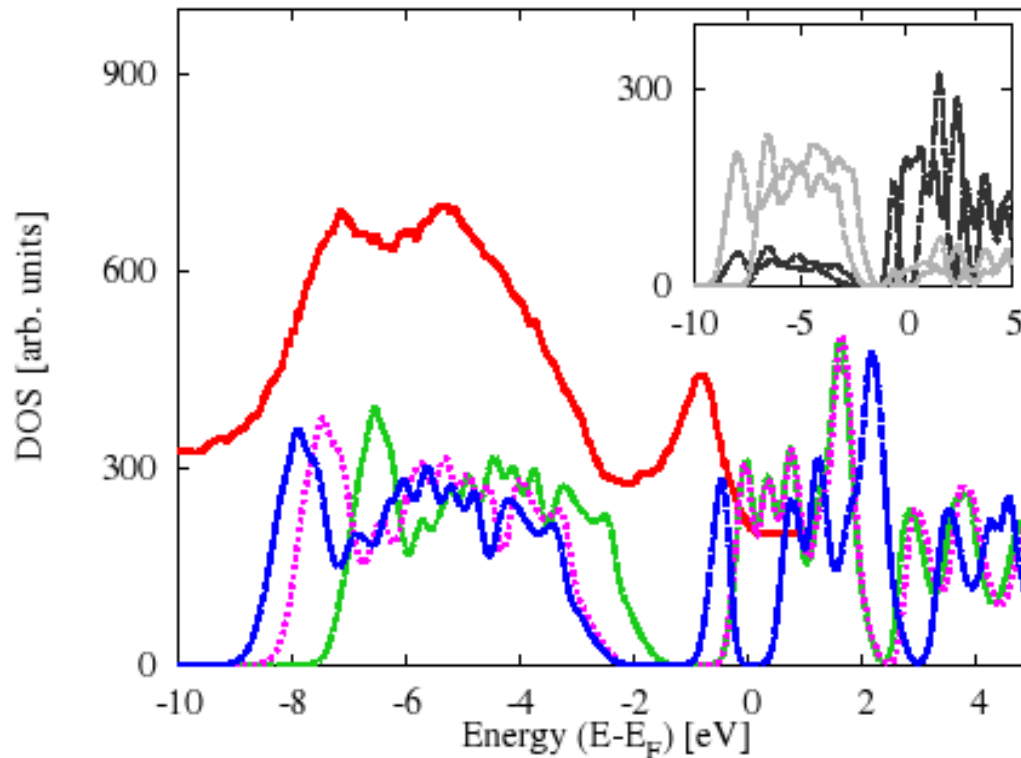


monoclinic

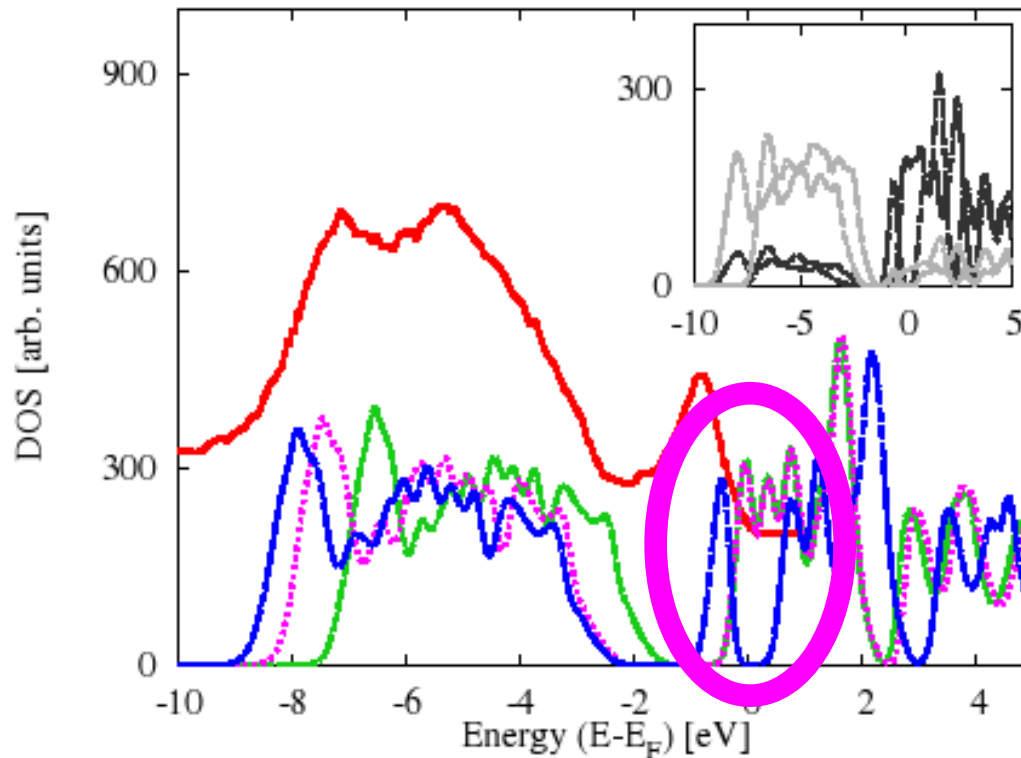




T. C. Koethe et al., Phys. Rev. Lett. 97, 116402 (2006).



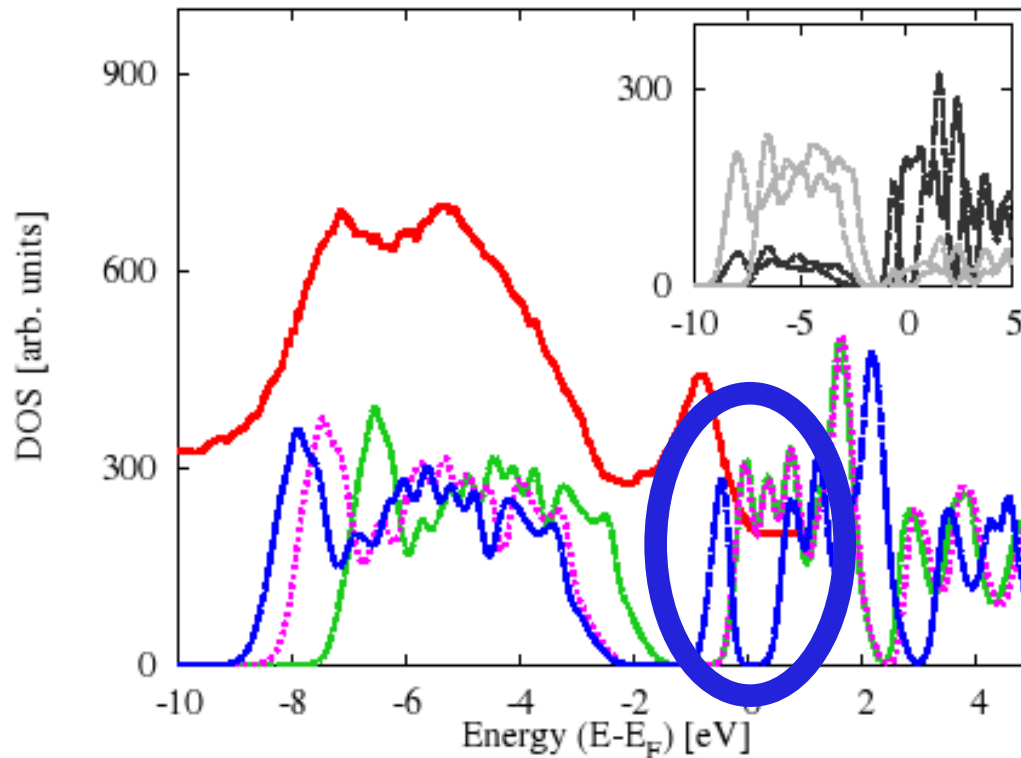
M. Gatti, F. Bruneval, V. Olevano and L. Reining,  
Phys. Rev. Lett. **99**, 266402 (2007)



**No  $G_0W_0$  gap**

M. C.

Phys. Rev. Lett. 99, 206402 (2007)



**Self consistency opens gap**

M. C.

Phys. Rev. Lett. 99, 200402 (2007)

# Away from the LDA starting point!

- S. V. Faleev, M. van Schilfgaarde, and T. Kotani, PRL 93, 126406 (2004).
- F. Bruneval, N. Vast, and L. Reining, Phys. Rev. B 74, 045102 (2006).
- F. Fuchs, et al., Phys. Rev. B 76, 115109 (2007).

# Away from the LDA starting point!

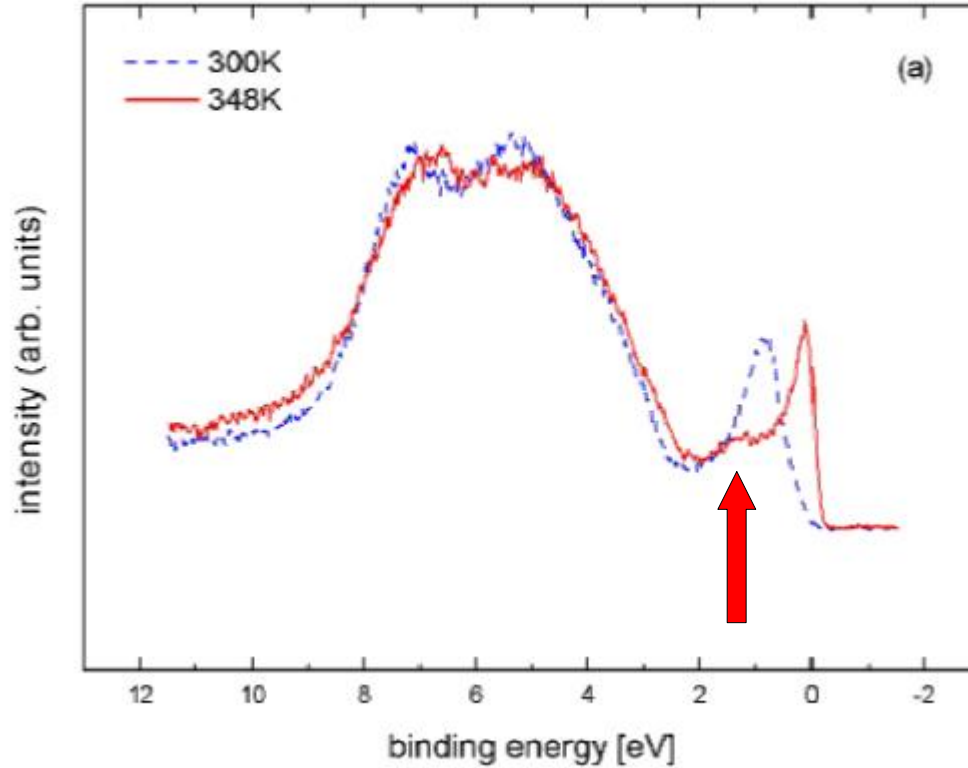
...of practical importance: example photovoltaics

Beyond Standard GW

## Quasiparticle energies within sc-GW for CIS

	CuInS <sub>2</sub>			
	DFT-LDA	G <sub>0</sub> W <sub>0</sub>	sc-GW	exp.
$E_g$	-0.11	0.28	1.48	1.54
In-(S,Se)	6.5	6.9	7.0	6.9
(Se,S) s band	12.4	13.0	13.6	12.0
In 4 d band	14.6	16.4	18.2	18.2

	CuInSe <sub>2</sub>			
	DFT-LDA	G <sub>0</sub> W <sub>0</sub>	sc-GW	exp.
$E_g$	-0.29	0.25	1.14	1.05 (+0.2)
In-(S,Se)	5.8	6.15	6.64	6.5
(Se,S) s band	12.6	12.9	13.6	13.0
In 4 d band	14.7	16.2	17.8	18.0



T. C. Koethe et al., Phys. Rev. Lett. 97, 116402 (2006).

# **PFlop:**

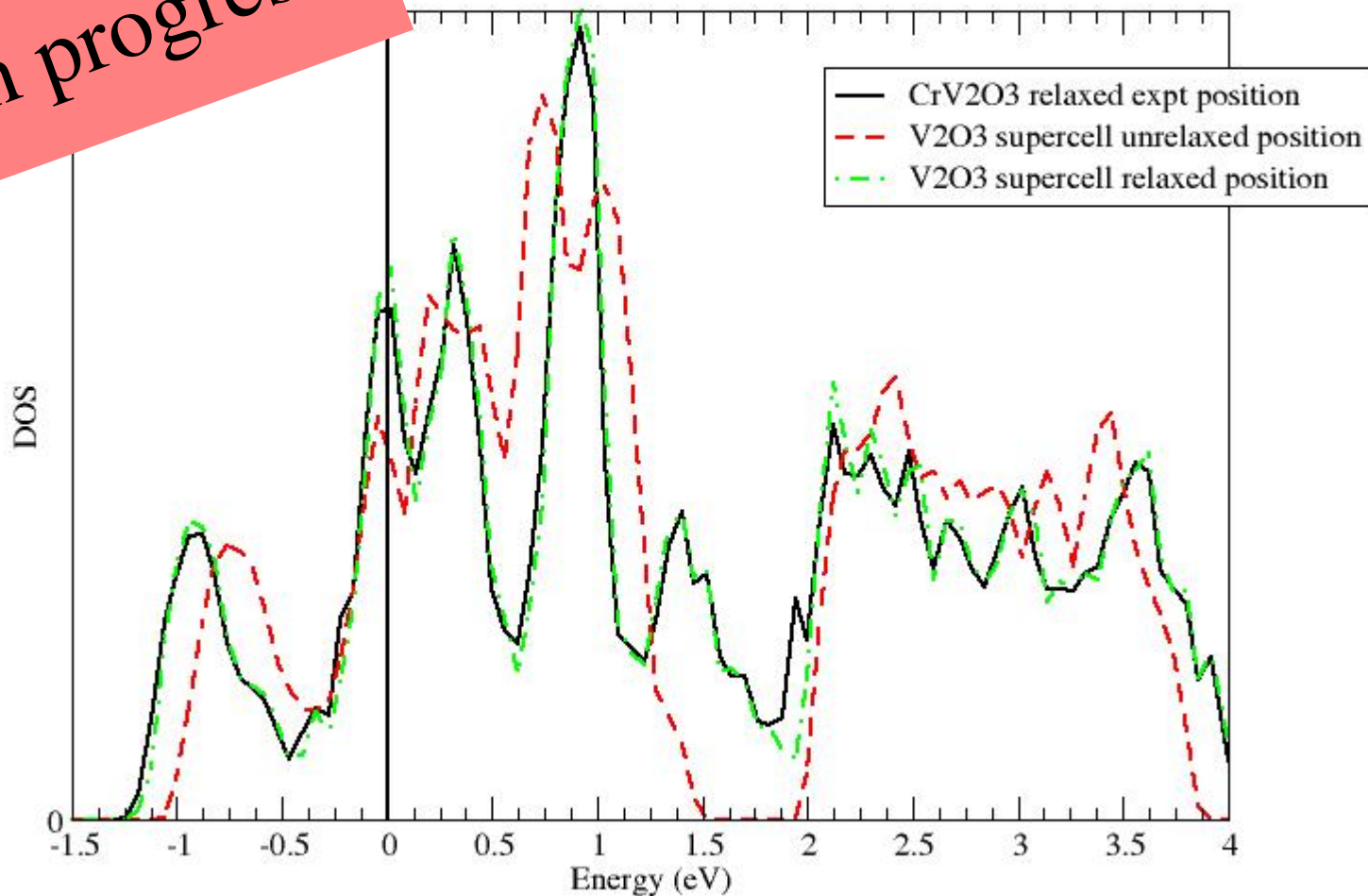
- spectral functions with good precision
- spectral functions beyond GW
- defects, doping





HF always large gap (12 eV) – scCOHSEX always metal

Work in progress!



# PFlop:

- doped material with advanced ground state calculation
- spectral functions GW
- beyond GW: better W
- beyond GW:  $\text{GW}\Gamma$ , e.g. self-screening correction

## Frontiers II: merge the 2 pictures:

$$\text{TDDFT } \chi = \chi_0 + \chi_0 [v + f_{xc}] \chi$$

$$\text{BSE } {}^4\chi = {}^4\chi_0 + {}^4\chi_0 [v + \delta\Sigma_{xc}/\delta G] {}^4\chi$$

(Waiting for PFlop...)

$$\text{TDDFT } \chi = \chi_0 + \chi_0 [v + f_{xc}] \chi$$

$$\text{BSE } {}^4\chi = {}^4\chi_0 + {}^4\chi_0 [v + \delta\Sigma_{xc}/\delta G] {}^4\chi$$

$$\text{TDDFT } \chi = \chi_0 + \chi_0 [v + f_{xc}] \chi$$

$$\text{BSE } {}^4\chi = {}^4\chi_0 + {}^4\chi_0 [v + \delta\Sigma_{xc}/\delta G] {}^4\chi$$

Introduce DF concept into MBPT

$$\text{TDDFT } \chi = \chi_0 + \chi_0 [v + f_{xc}] \chi$$

$$\text{BSE } {}^4\chi = {}^4\chi_0 + {}^4\chi_0 [v + \delta\Sigma_{xc}/\delta G] {}^4\chi$$

Introduce DF concept into MBPT

$$\delta\Sigma_{xc}(1,2)/\delta G(3,4) \rightarrow \delta\Sigma_{xc}(1,2)/\delta\rho(3)$$

This leads directly to equation for  $\chi$

$$\delta V_{xc}/\delta\rho$$

TDDFT

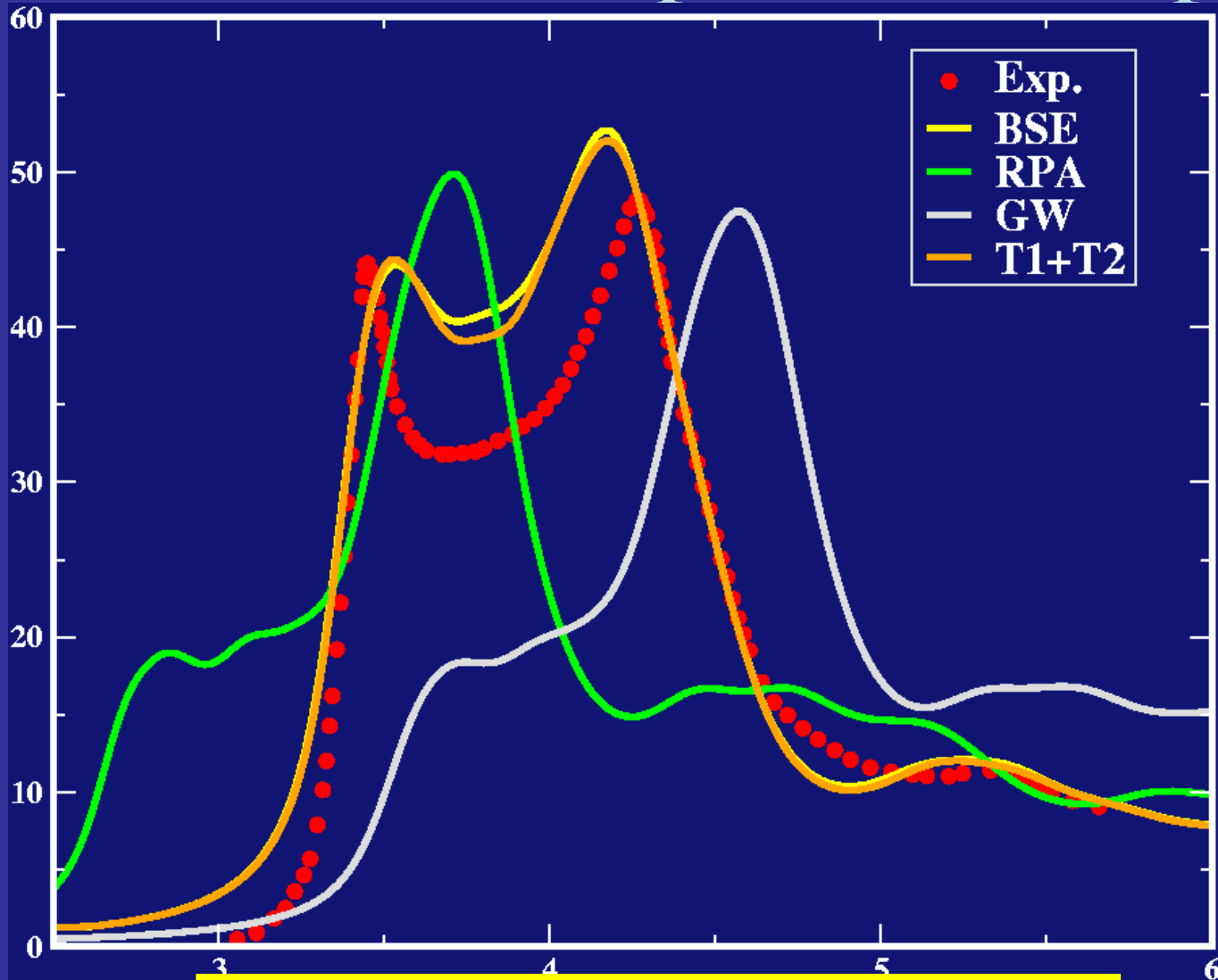
$$\chi = \chi_0 + \chi_0 [ v + f_{xc} ] \chi$$

$$\delta V_{xc}/\delta\rho$$

$$\text{TDLDA: } \delta V_{xc}^{\text{LDA}}(r,t)/\delta\rho(r',t') = \delta(r-r')\delta(t-t')dV_{xc}/d\rho$$

$$f_{xc} \text{ from BSE: } \sim \text{shift} + \chi_0^{-1}GGWGG\chi_0^{-1}$$

# Silicon: Optical Absorption



F. Sottile PhD thesis

Hanke and Sham

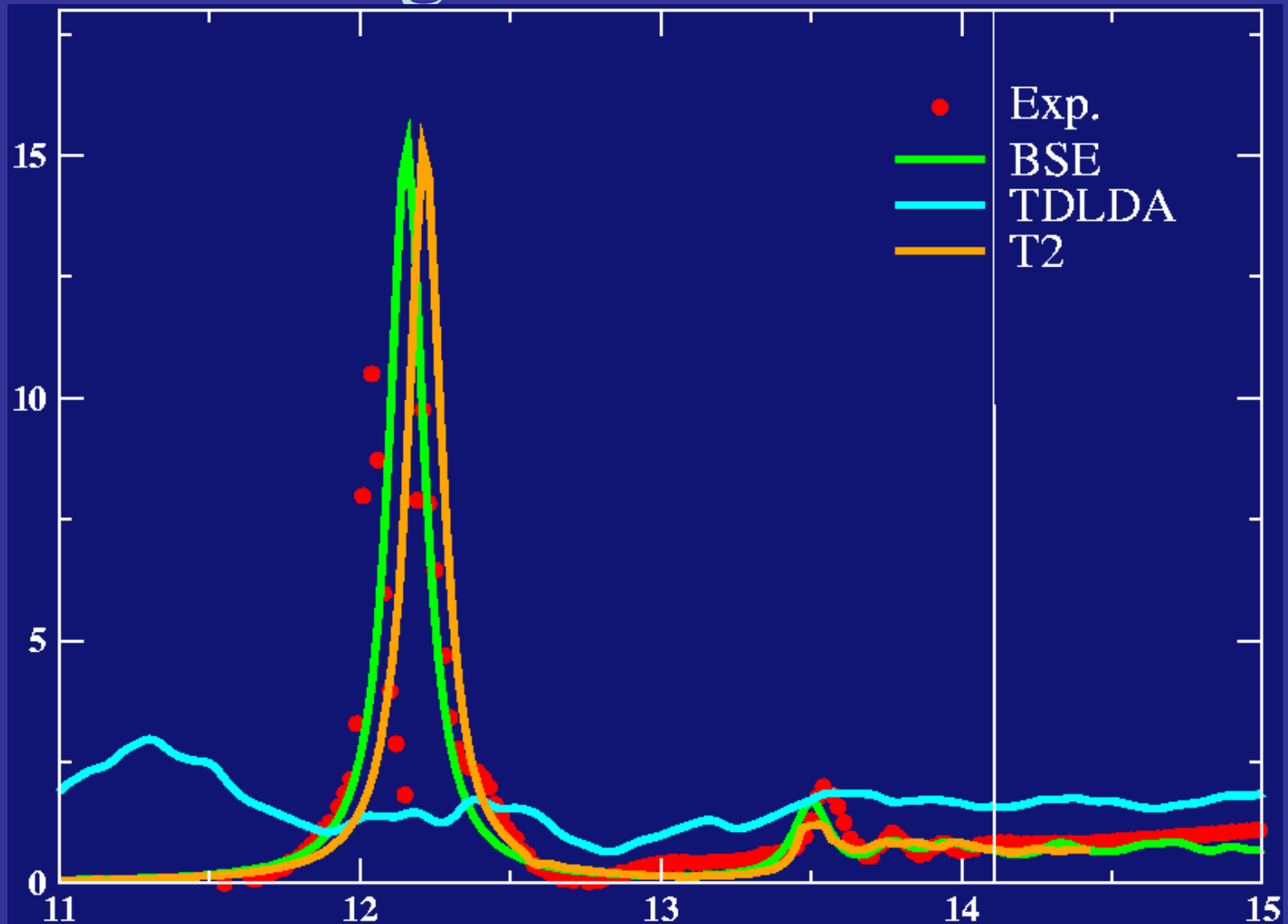
Albrecht et al.

Benedict et al.

BSE and TDDFT: nice result



# Solid Argon: Bound Excitons



- Francesco Sottile, PhD thesis (2003)

# TDDFT including excitons:

Reining et al, PRL 88, 066404 (2002);

Sottile PhD thesis and Sottile et al, PRL 91, 056402 (2003)

Bruneval PhD thesis and

Bruneval, Sottile, Olevano, Del Sole, Reining, PRL (2005)

Adragna PhD thesis and Adragna et al, PRB 68, 165108 (2003),

Marini et al PRL 91, 256402 (2003)

Stubner, Tokatly, Pankratov, PRB 70, 245119 (2004)

and Ref. therein

von Barth, Dahlen, van Leeuwen, Stefanucci,

PRB 72, 235109 (2005)

Still:

- calculation of full spectral function delicate  
+ long
- satellite masked by absence of vertex in  $\Sigma$   
( $W$  screens classical charge)

## Frontiers III: compromise?

$V_{xc}([\rho]; r) \delta(r-r')$   $\rightarrow$   $\rho(r)$  *no spectrum, no gap*

$V_{xc}([\rho]; r, t) \delta(r-r') \delta(t-t')$   $\rightarrow$   $\rho(r, t)$  *spectrum, optical gap*

$\Sigma(r, r', t, t')$   $\rightarrow$   $G(r, r', t, t')$  *QP gap, ARPES, propagation, ...*

## Frontiers III: compromise?

$V_{xc}([\rho]; r) \delta(r-r')$   $\rightarrow \rho(r)$  *no spectrum, no gap*

$V_{xc}([\rho]; r, t) \delta(r-r') \delta(t-t')$   $\rightarrow \rho(r, t)$  *spectrum, optical gap*

$\Sigma(r, r', t, t')$   $\rightarrow G(r, r', t, t')$  *QP gap, ARPES, propagation, ...*

$\rightarrow \text{Im}[G(r, r, \omega)]$  *QP gap, XPS*

## Frontiers III: compromise?

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$V_{xc}([\rho]; r, t)\delta(r-r')\delta(t-t')$   $\rightarrow$   $\rho(r, t)$  *spectrum, optical gap*

$\Sigma(r, r', t, t')$   $\rightarrow$   $G(r, r', t, t')$  *QP gap, ARPES, propagation,...*

$V_{SF}(r, \omega)$

$\rightarrow \text{Im}[G(r, r, \omega)]$  *QP gap, XPS*

$V_{SF}(r, \omega)$  *is real and local in space*

## 6) Summary

$\delta V$  makes.....

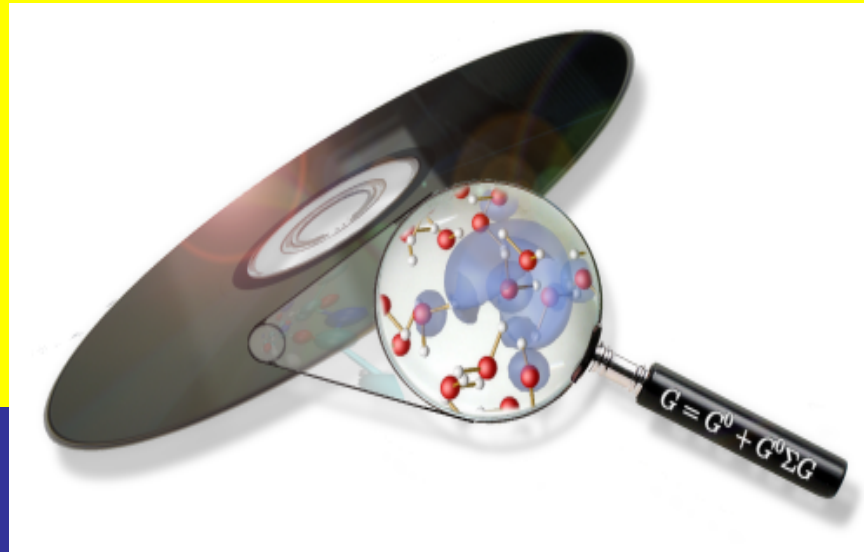
- \* crystal local field effects
- \* anomalous angular dependence of IXS spectra
- \* excitons
- \* satellites
- \* .....

Most profitable to take different points of view !!!!

## 6) Summary

We need.....

- \* crystal local field effects
- \* anomalous angular dependence of IXS spectra
- \* excitons
- \* satellites
- \* .....

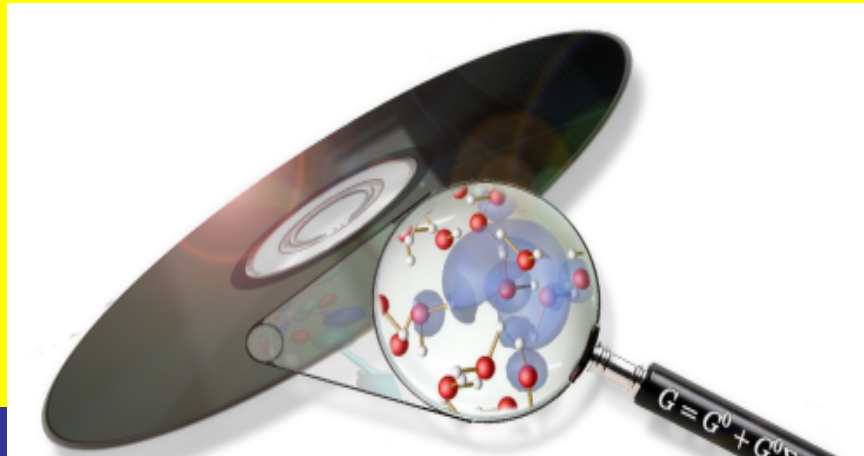




## 6) Summary

We need.....

- \* crystal local field effects
- \* anomalous angular dependence of IXS spectra
- \* excitons
- \* satellites
- \* .....



More thoughts and more computers !!!!

# Ab initio calculations of electronic response: ingredients, results and challenges

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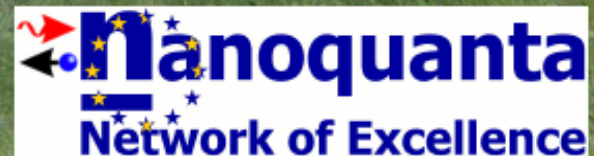
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Experimentalists





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