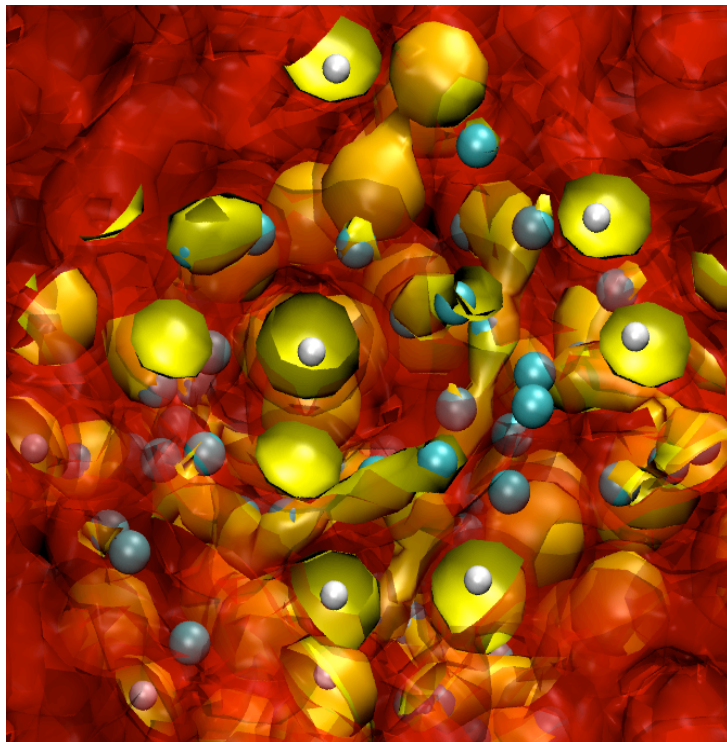


First-Principles Simulations in Planetary Science



Burkhard Militzer

University of California, Berkeley

June 20, 2008

<http://militzer.berkeley.edu>

Outline and Acknowledgements

Hydrogen-
helium
calculation:

Path Integral Monte Carlo (PIMC)

Density Functional Theory

Jan Vorberger (University of Warwick, UK)

Isaac Tamblyn (Dalhousie Univ., Halifax)

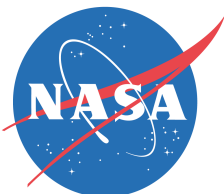
Modeling of Jovian planets:

Bill Hubbard (LPL, University of Arizona, Tucson)

David Stevenson (Caltech)

QMC of solid helium:

Saad Khairallah (UC Berkeley)



Supported by [NASA](#) and [NSF](#).

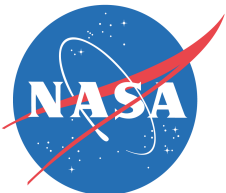


Outline and Acknowledgements

Main points:

I. Jupiter has a solid core of 16 Earth masses.

II. It does not rotate as a “solid” body
⇒ differential rotation on cylinders.



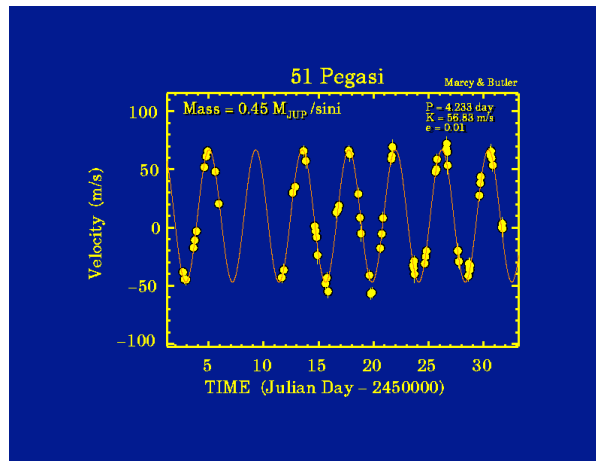
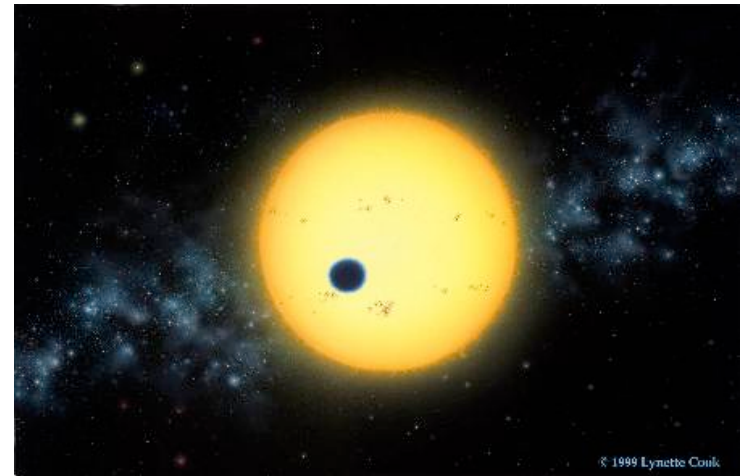
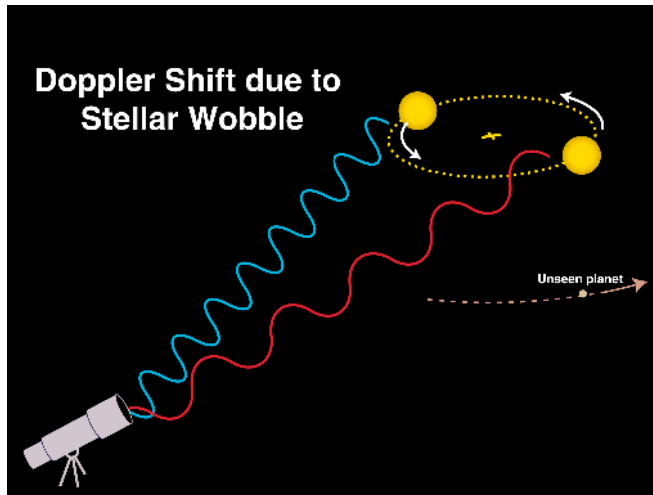
Supported by [NASA](#) and [NSF](#).



Detection techniques for **extrasolar planets**: **radial velocity** technique, **transient** method, ...

270+ planets found with radial velocity meas.

14 planets seen with transient technique



First planet detected:

Mayor & Queloz, Nature 378 (1995) 355
(Geneva Observatory)

Orbital period: 4.23 days !

$M \sin(i) = 0.46$

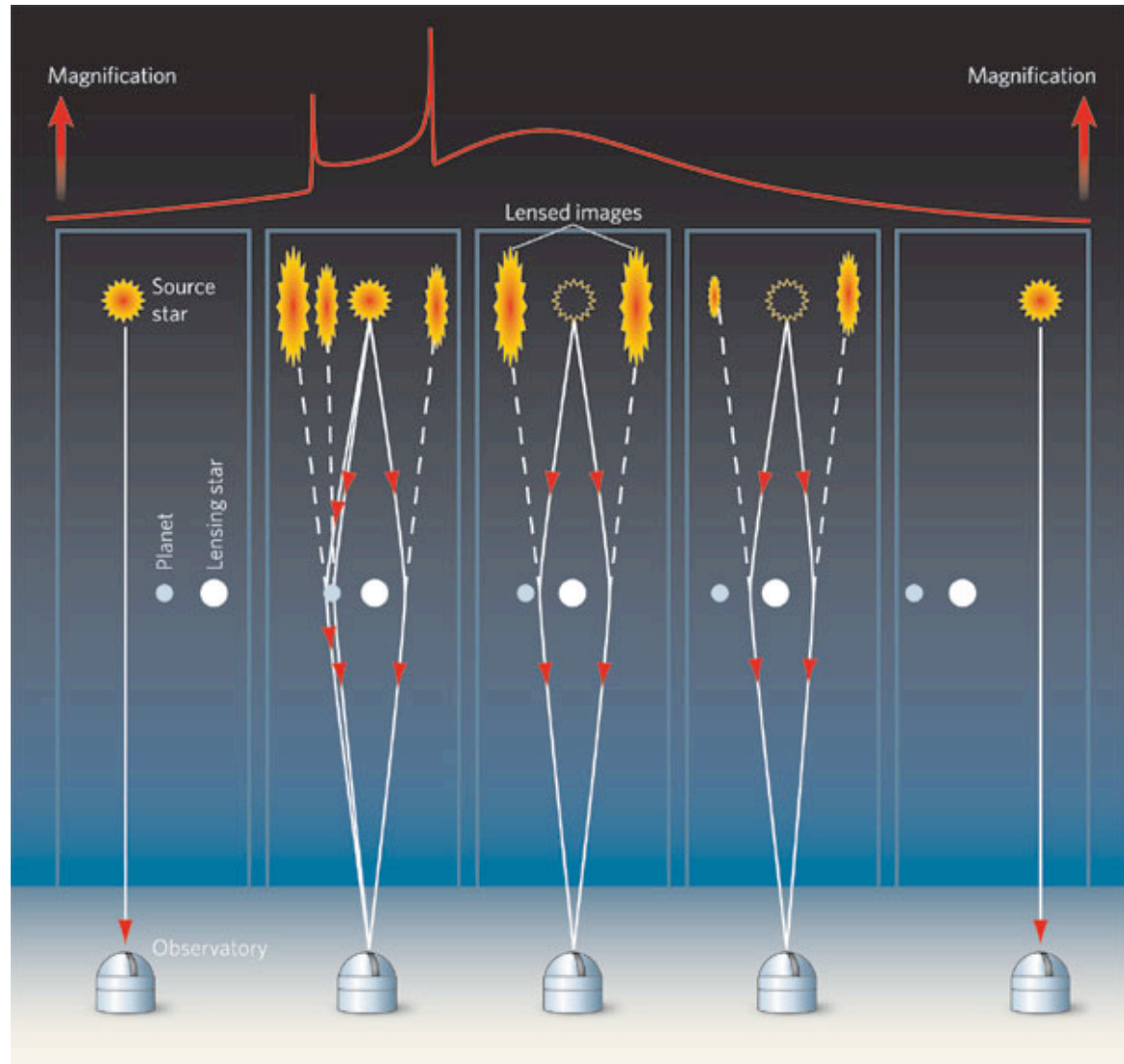
$a = 0.05 \text{ AU}$

2006: **Third technique** finds extrasolar planets: observation of **gravitational microlensing events**

So far, **4** planets found by observation of gravitational microlensing events.

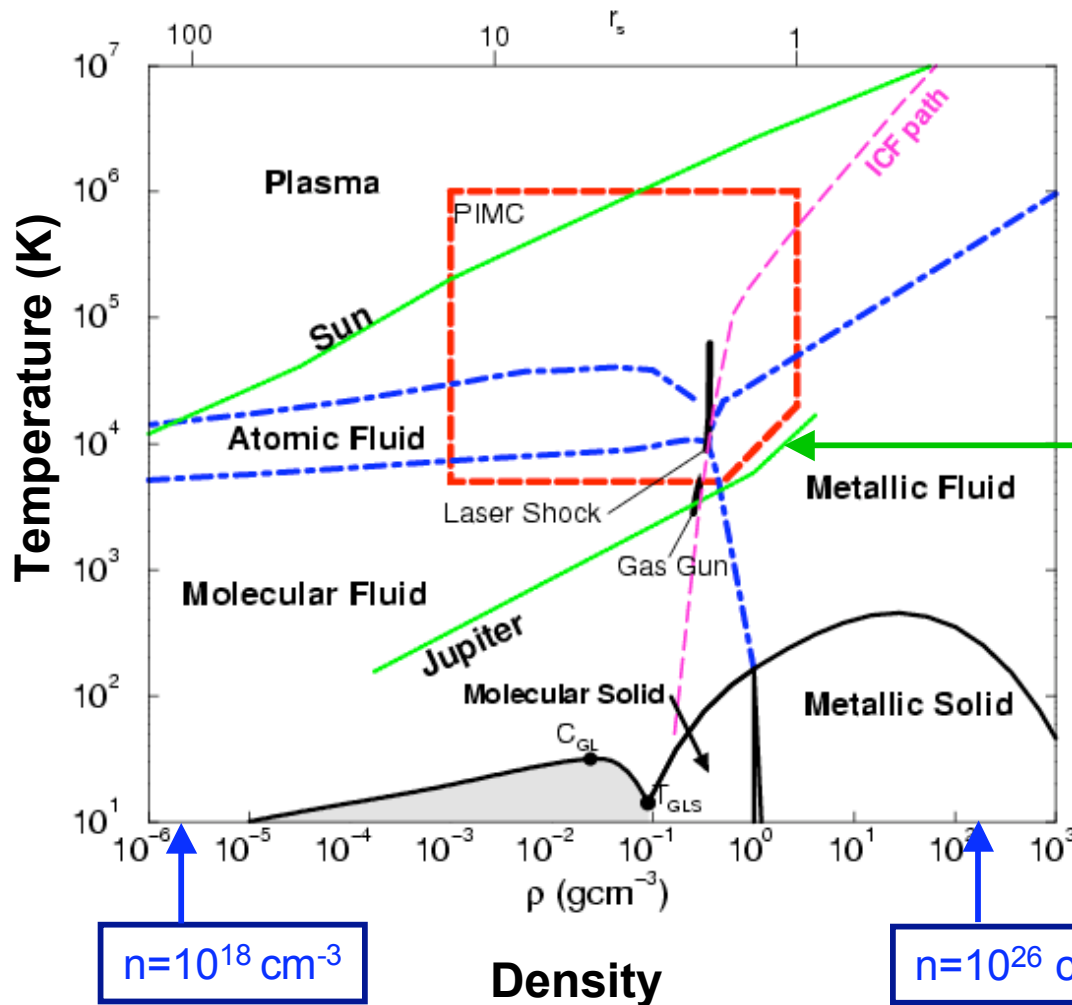
J.P. Beaulieu et al, Nature (2006).

Detected planet well below the Doppler detection limit.

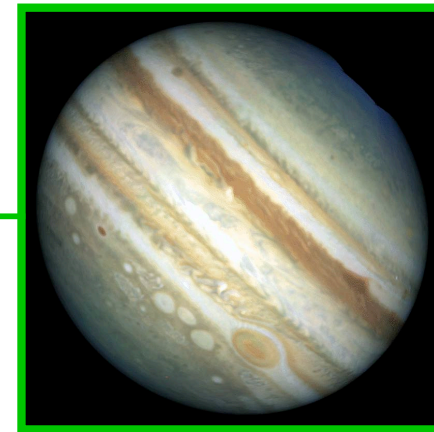


D. Queloz, Nature, New & Views (2006).

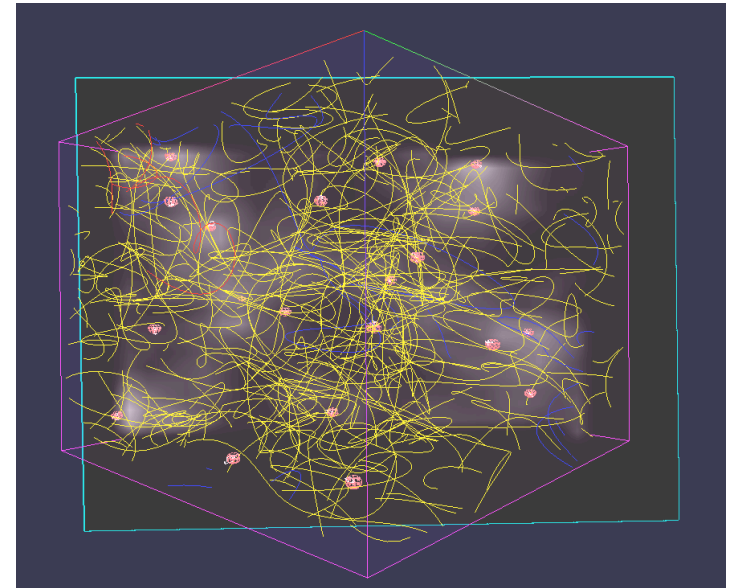
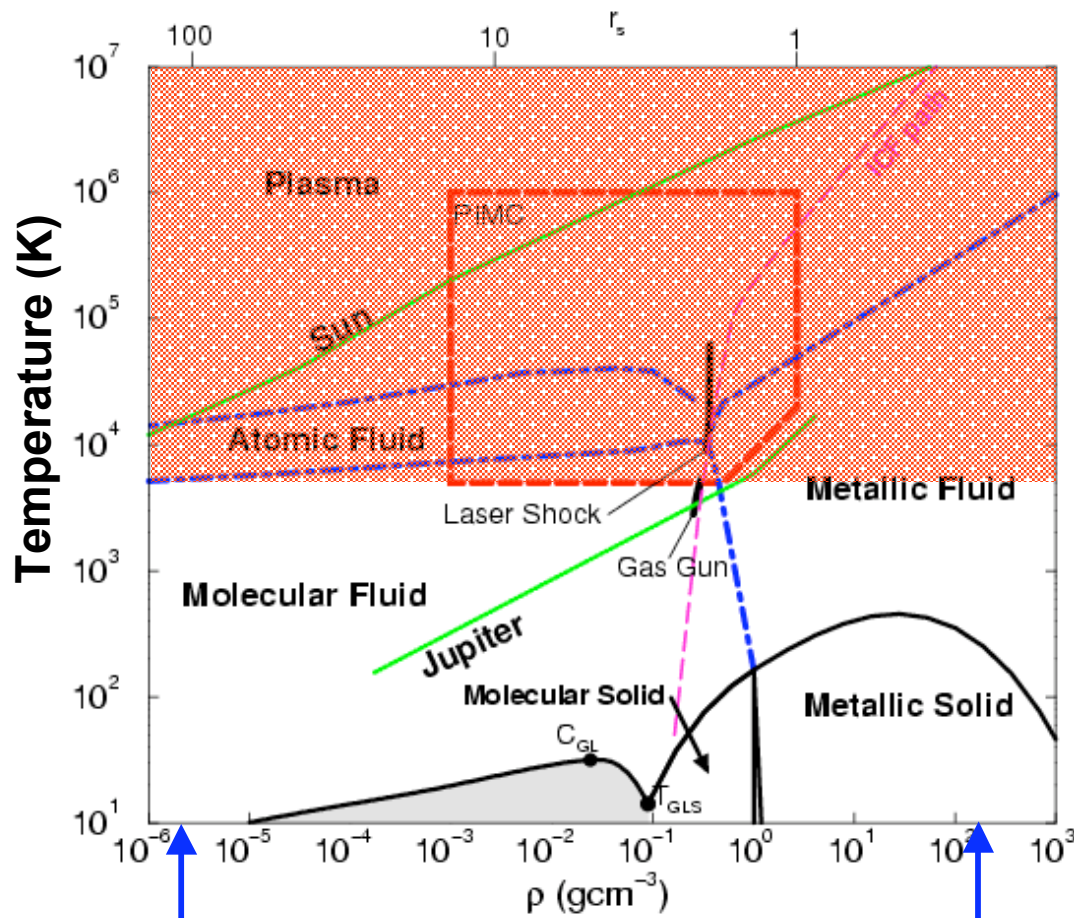
Focus: Characterization of the Interior of Solar and Extrasolar Giant Planets



Solar GP: Jupiter, Saturn



1) Path integral Monte Carlo for $T > 5000\text{K}$

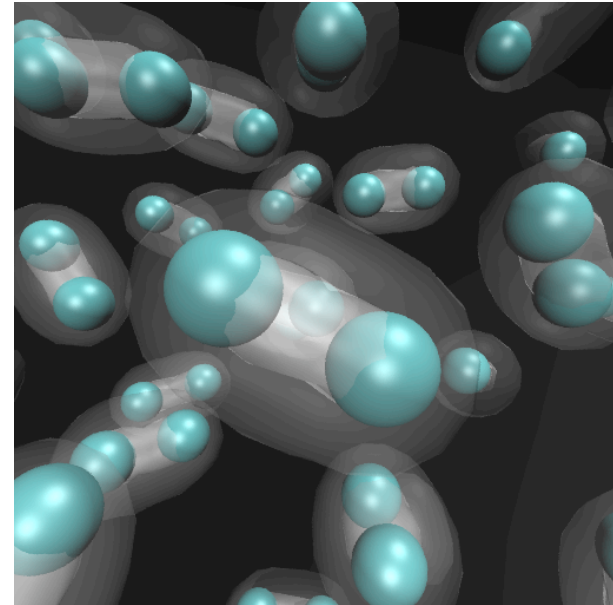
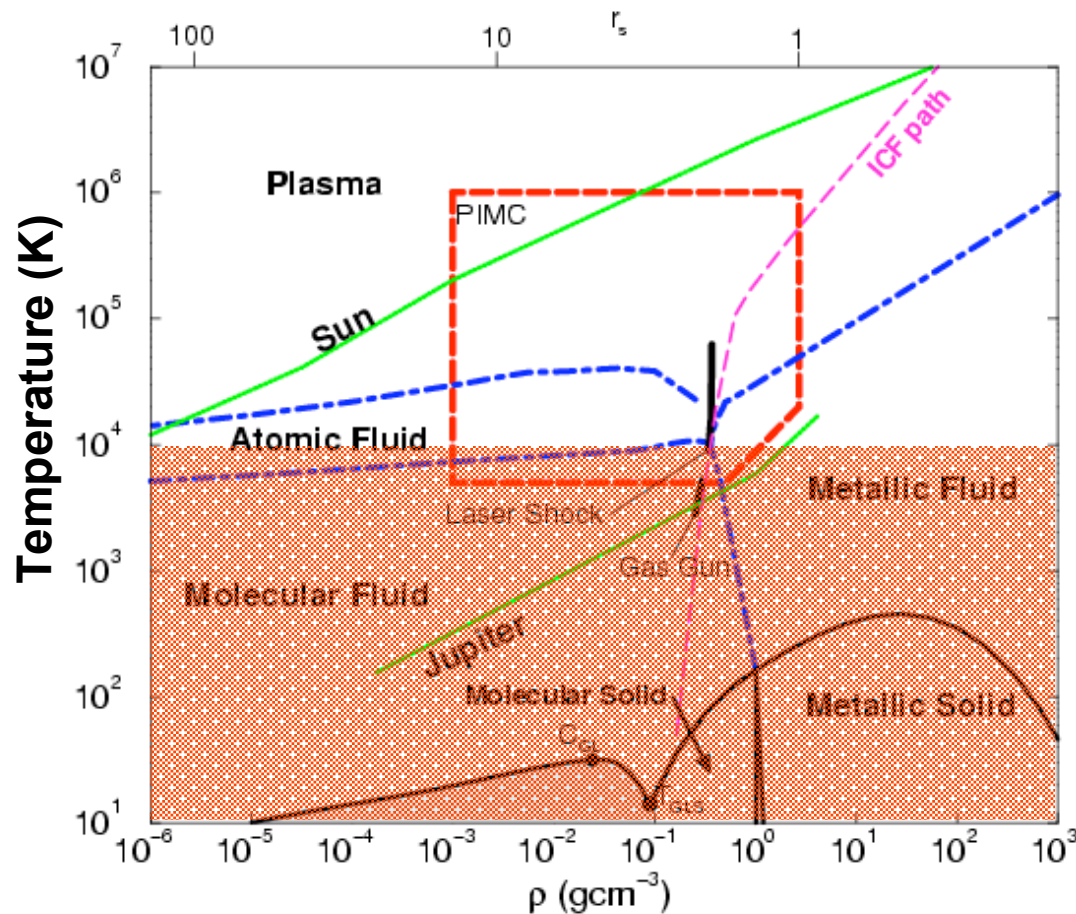


PIMC applicable at:
 $T > 5000\text{K}$

$n = 10^{18} \text{ cm}^{-3}$

$n = 10^{26} \text{ cm}^{-3}$

- 1) Path integral Monte Carlo for $T > 5000\text{K}$
- 2) Density functional molecular dynamics below

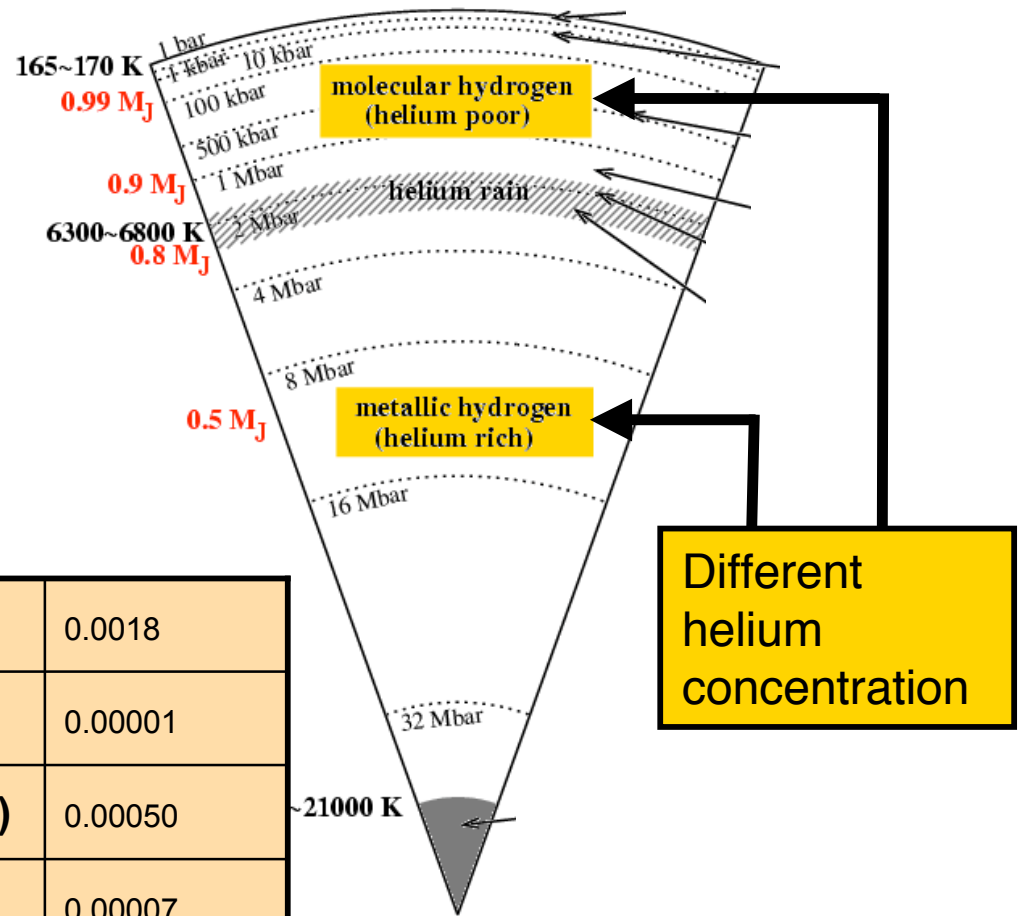
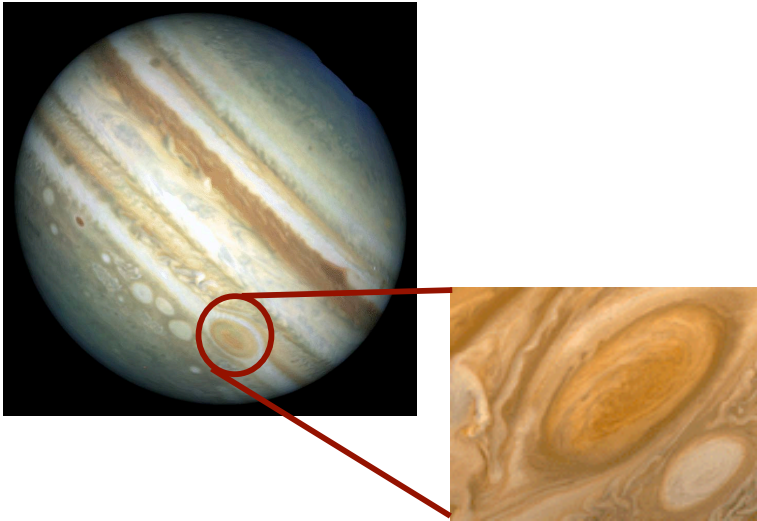


Born-Oppenheimer approx.
MD with classical nuclei:

$$\mathbf{F} = m \mathbf{a}$$

Forces derived DFT with electrons in the instantaneous ground state.

Previous Jupiter Models with 3 Layers

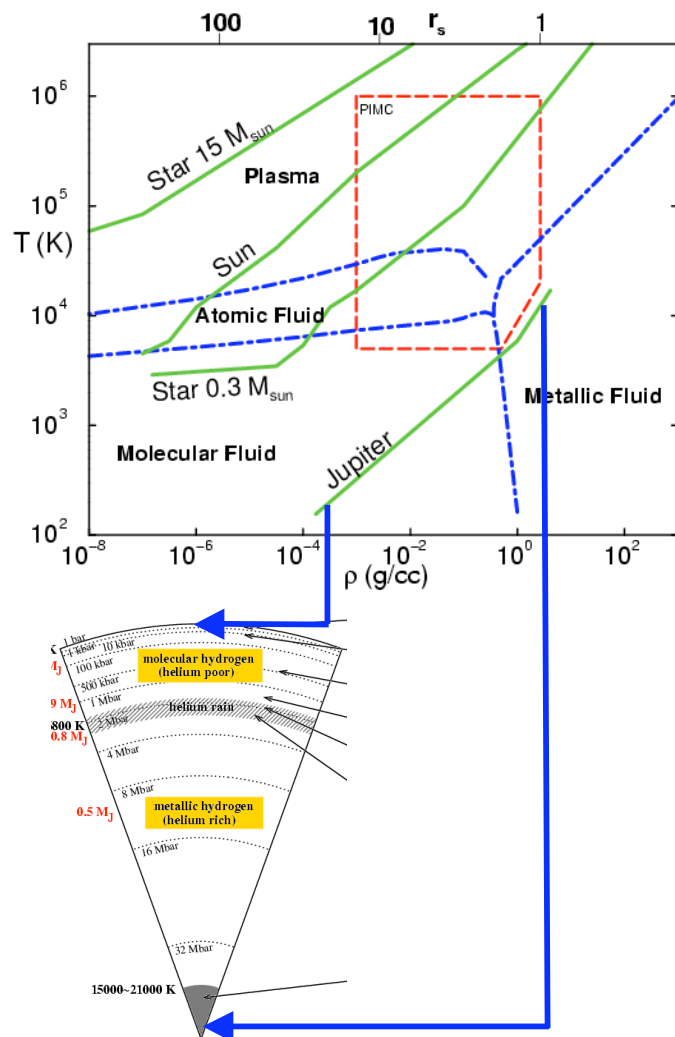


Composition on the surface (solar):

H	0.742	0.736	Ne	< 0.0002	0.0018
He	0.231(4)	0.249	P	< 0.00007	0.00001
C	0.009(2)	0.0029	S	0.00091(6)	0.00050
N	< 0.012	0.00085	Ar	< 0.00015	0.00007
O	< 0.0035	0.0057	“Z”	0.027	0.015

Guillot et al. (Jupiter book, 2002, chap.3)

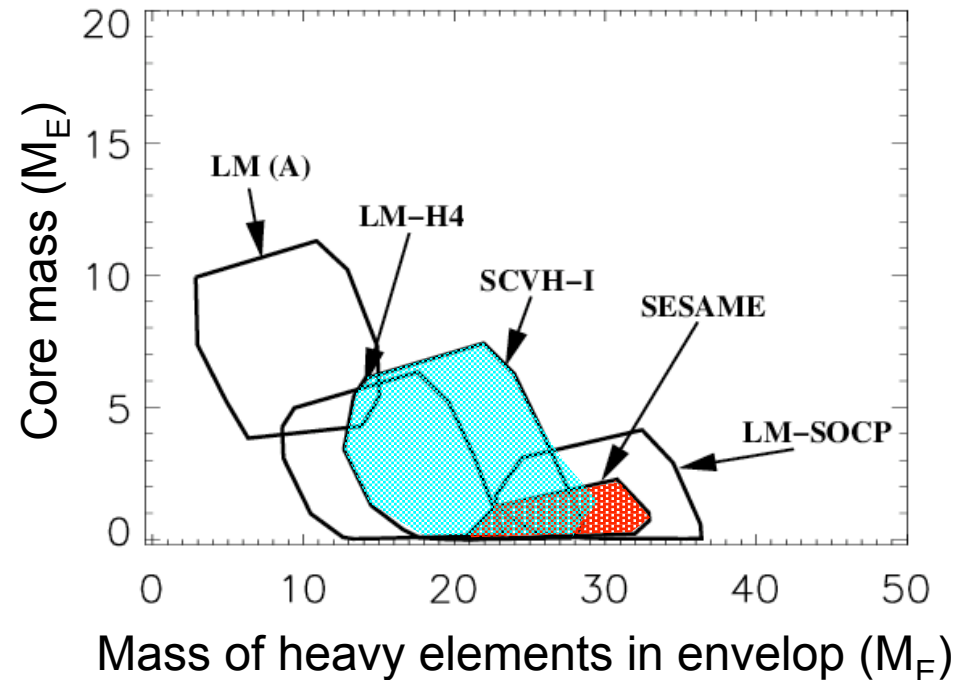
T. Guillot's model: Uncertainties in EOS do not allow to determine if Jupiter has a rocky core



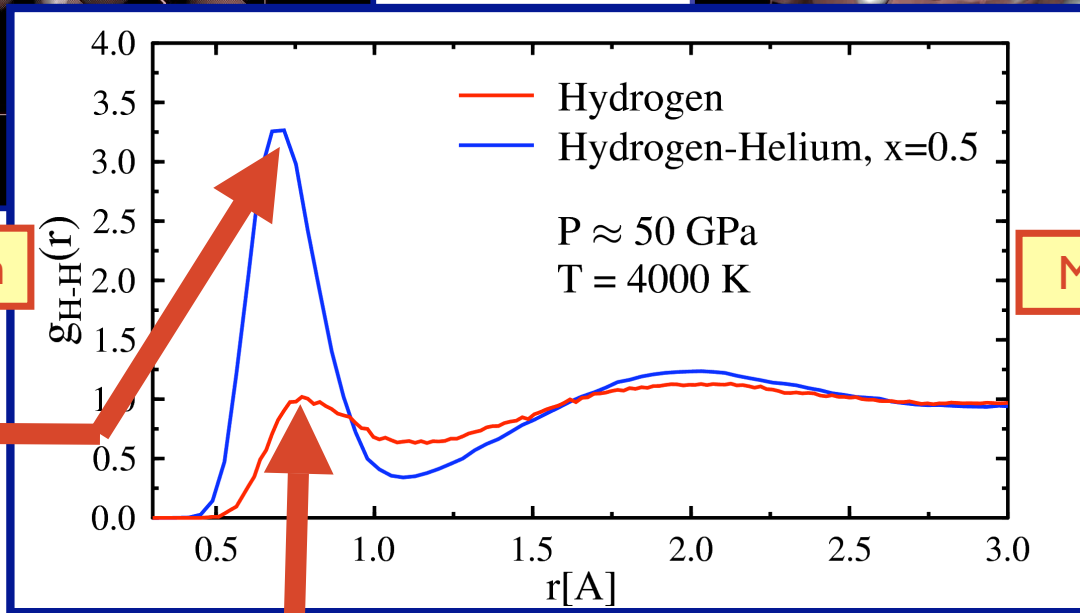
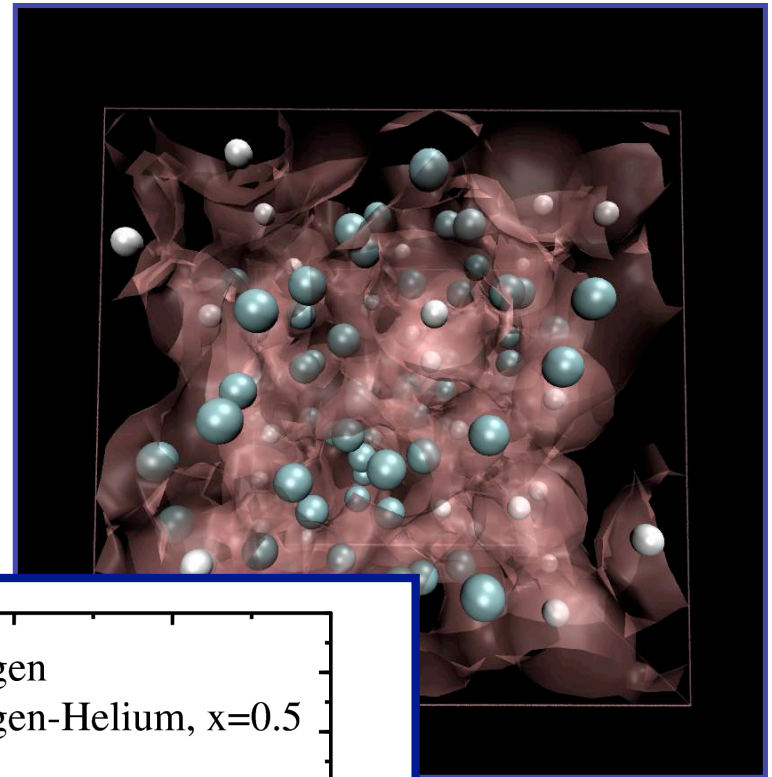
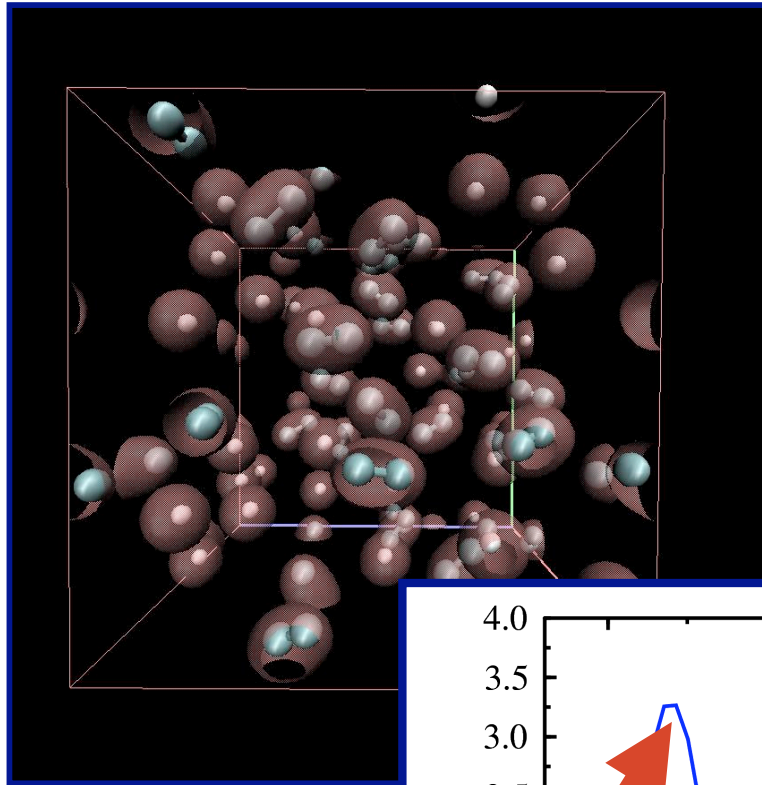
T. Guillot's three layer model is based on

- 1) Hydrogen-helium EOS
- 2) Surface composition
- 3) Gravitational moments inferred from fly-by trajectories (Cassini mission)

Parameters like **core mass** or amount of **heavier**



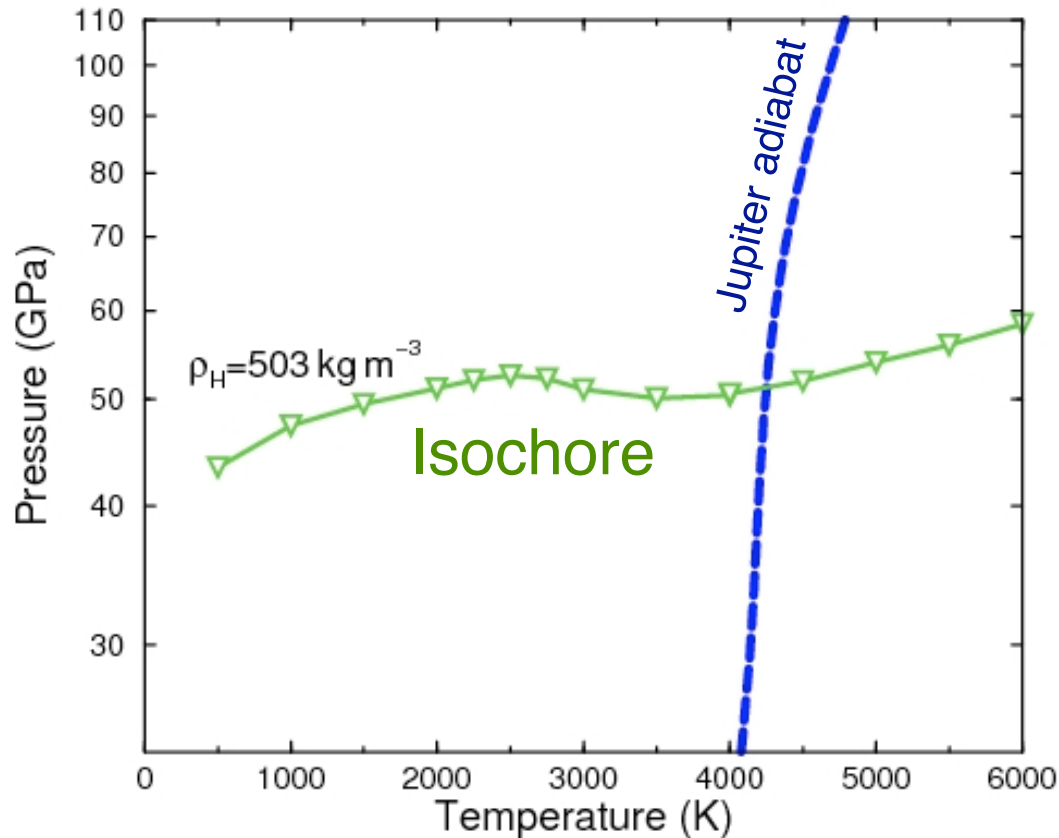
Simulations show that helium stabilizes H₂ molecules



Molecular hydrogen

Metallic hydrogen

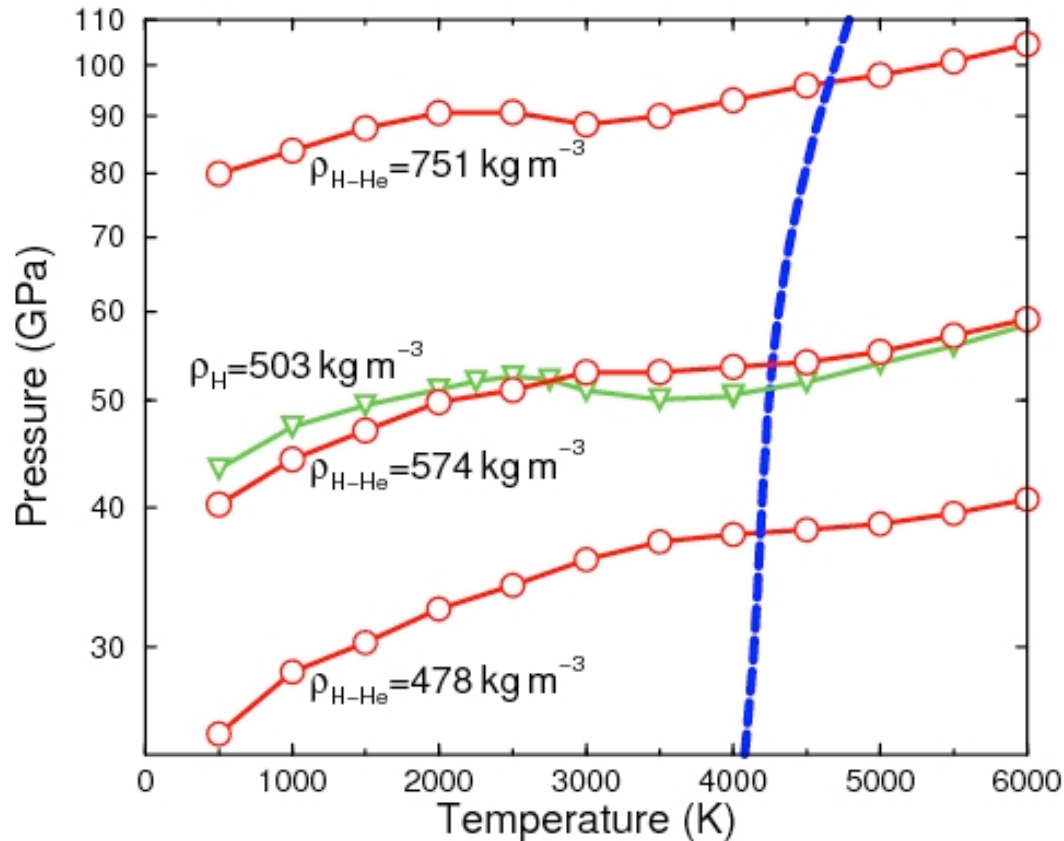
Molecular-to-metallic transition in fluid hydrogen studied with DFT simulations



- Molecular to atomic transition is **continuous** as function of T (must use BOMD not CPMD)
- Negative $dP/dT|_V < 0$ region for pure hydrogen

J. Vorberger, I. Tamblyn, B. Militzer, S. Bonev, "Hydrogen-helium mixtures in the interior of giant planets", *Phys. Rev. B* **75** (2007) 024206.

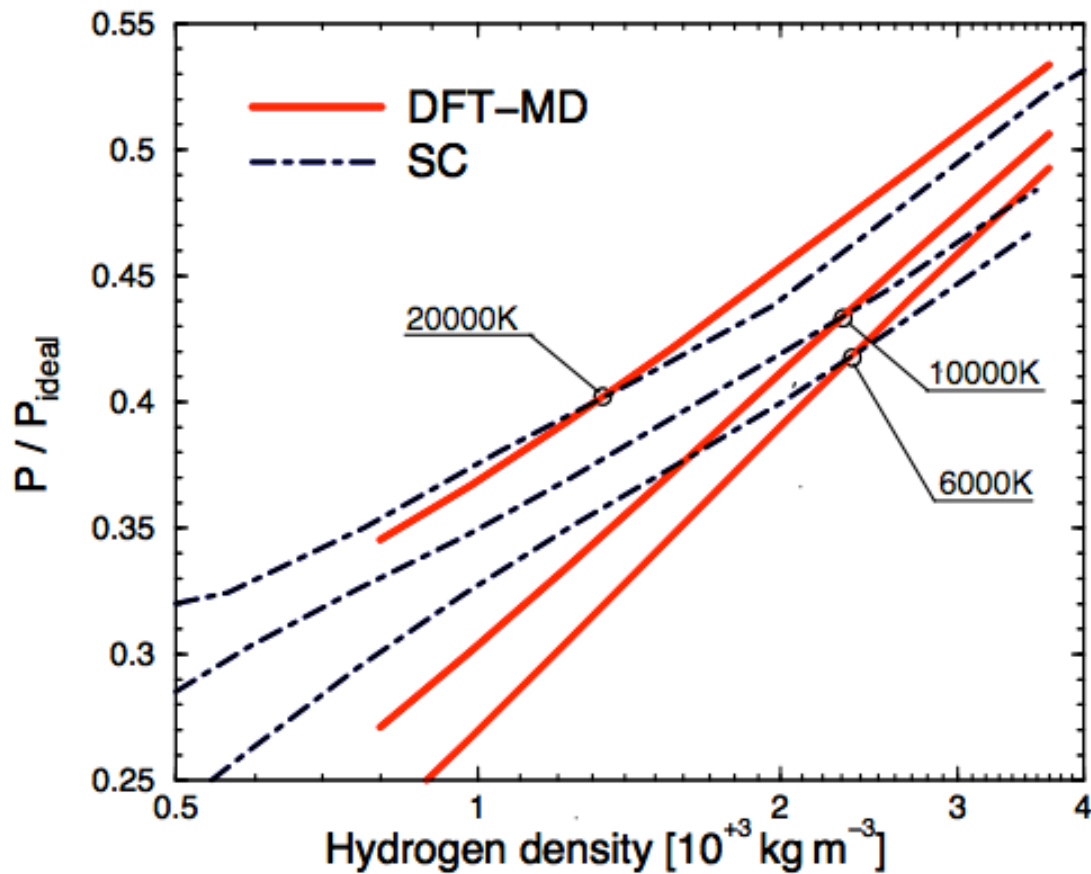
Molecular-to-metallic transition in fluid hydrogen studied with DFT simulations



- Molecular to atomic transition is **continuous** as function of T (must use BOMD not CPMD)
- Negative $dP/dT|_V < 0$ region for pure hydrogen
- **No** such region: H-He mixtures
- Jupiter envelope is isentropic, fully convective and of constant chemical composition.

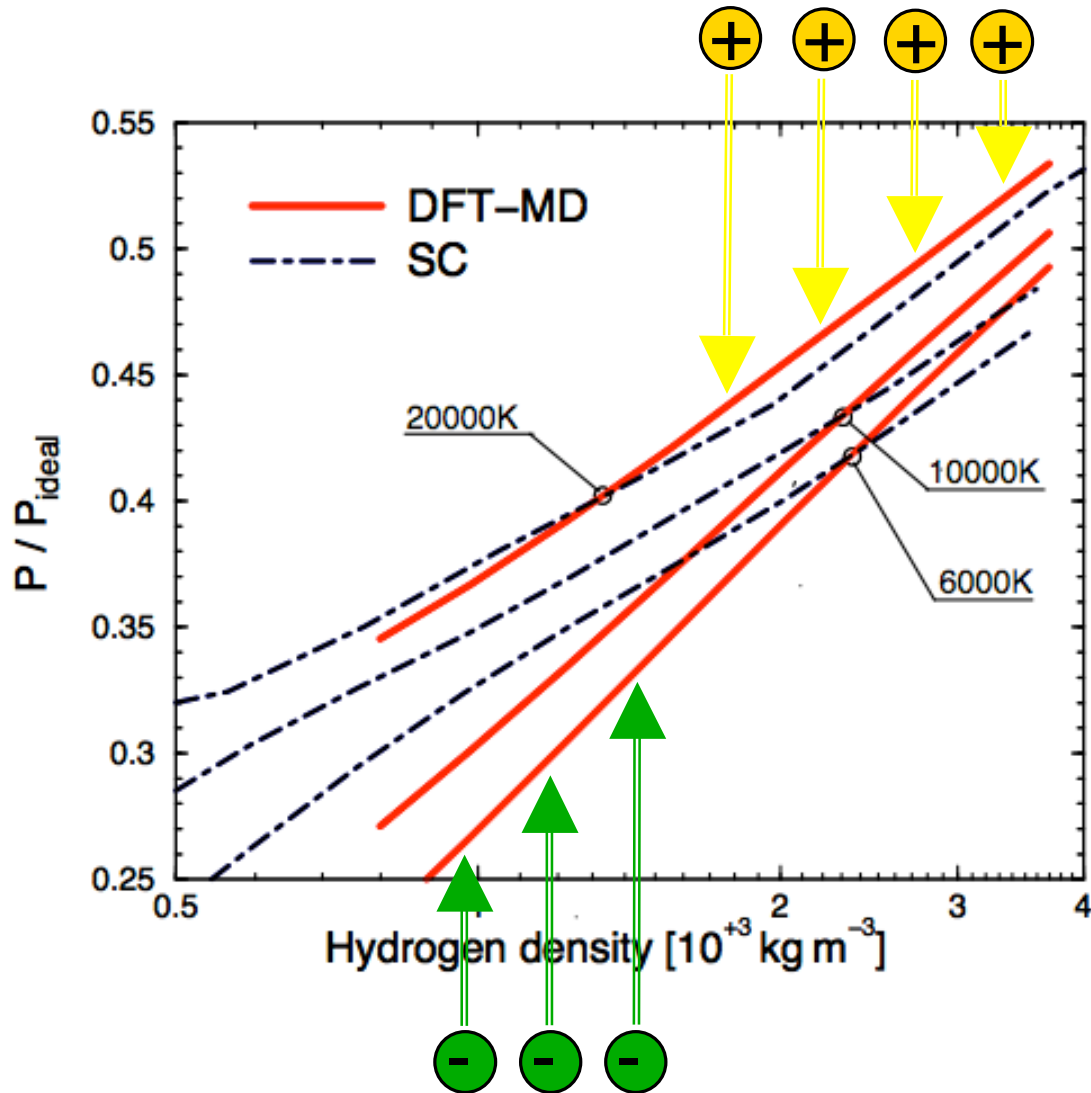
J. Vorberger, I. Tamblyn, B. Militzer, S. Bonev, "Hydrogen-helium mixtures in the interior of giant planets", *Phys. Rev. B* **75** (2007) 024206.

Comparison of first-principles EOS with analytical Saumon-Chabrier-Van Horn model



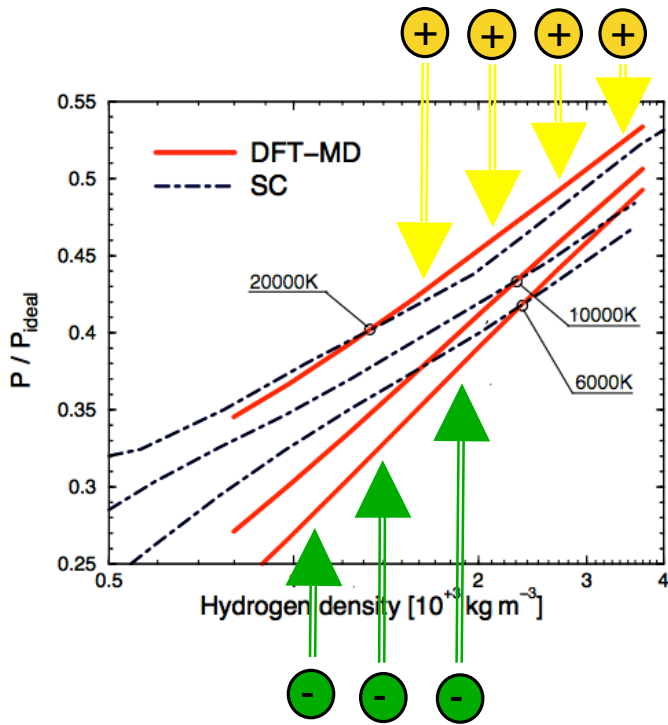
The most important difference were found in the regime of **metallic hydrogen** where little experimental EOS data are available.

Comparison of first-principles EOS with analytical Saumon-Chabrier-Van Horn model



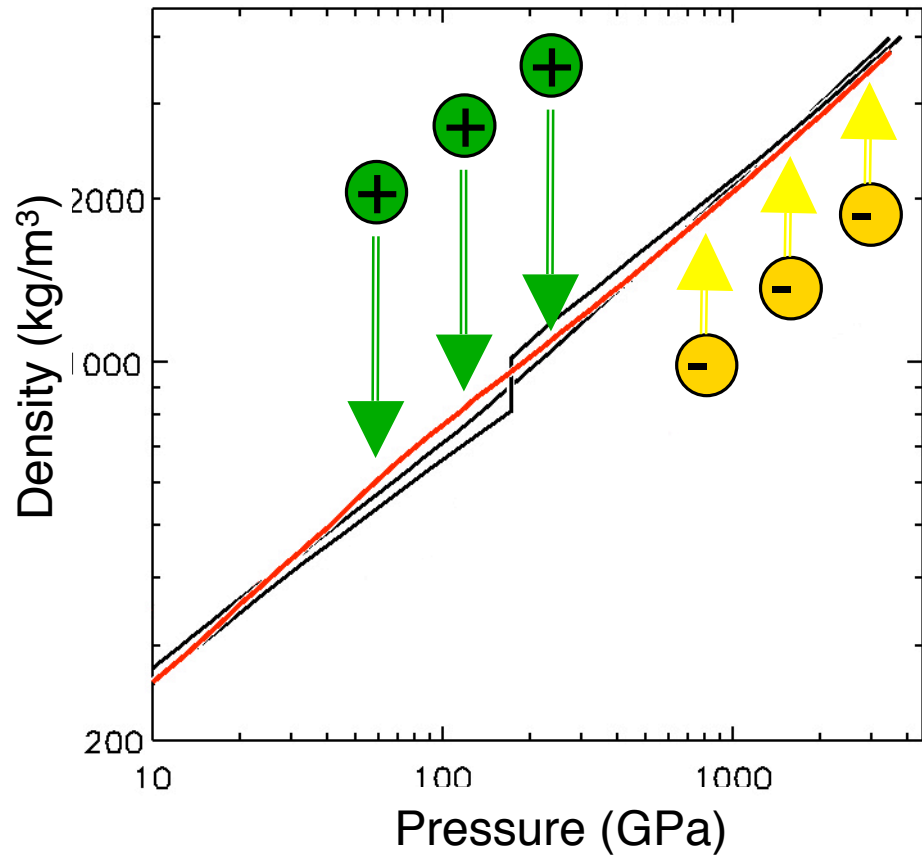
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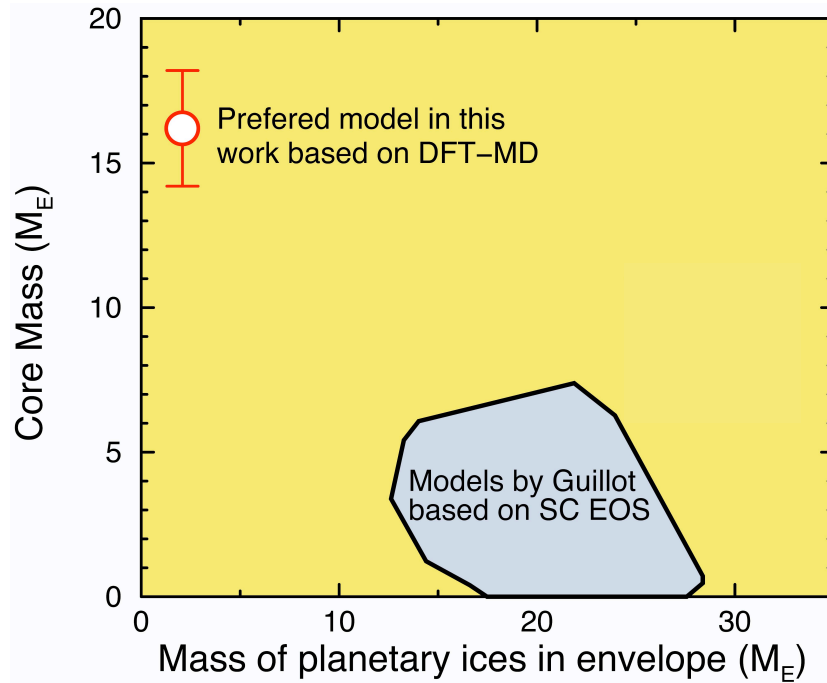


Direct EOS comparison

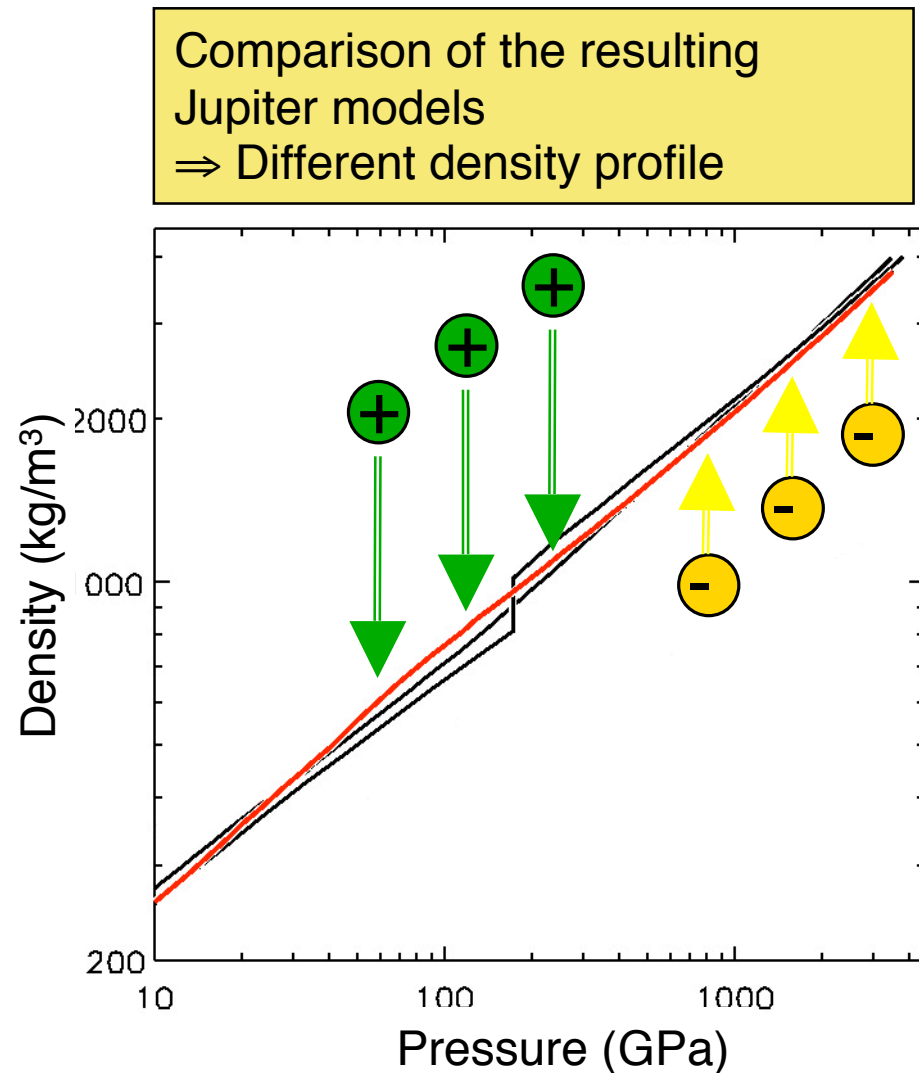
Comparison of the resulting Jupiter models
⇒ Different density profile



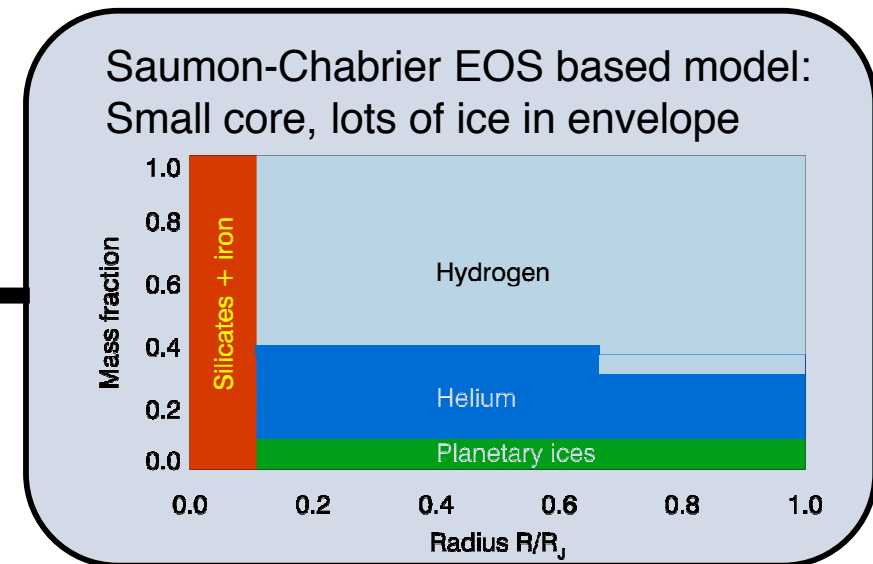
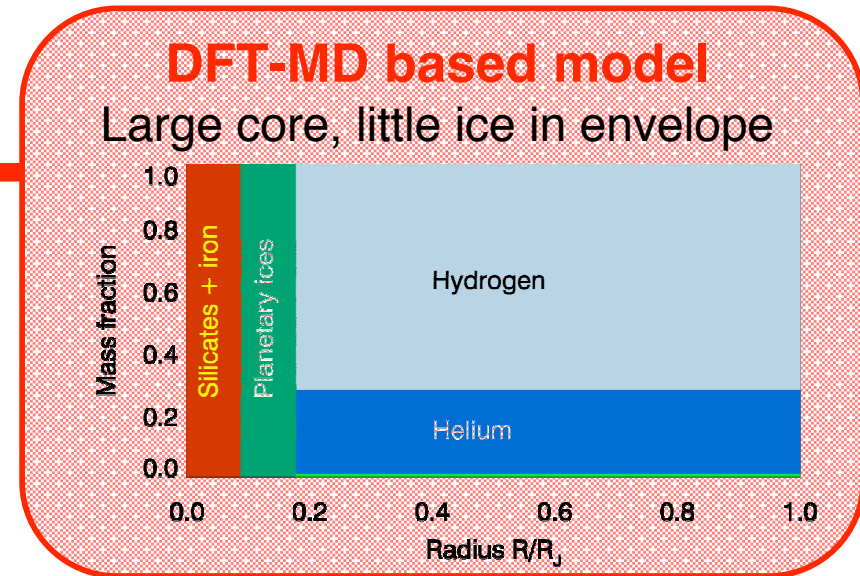
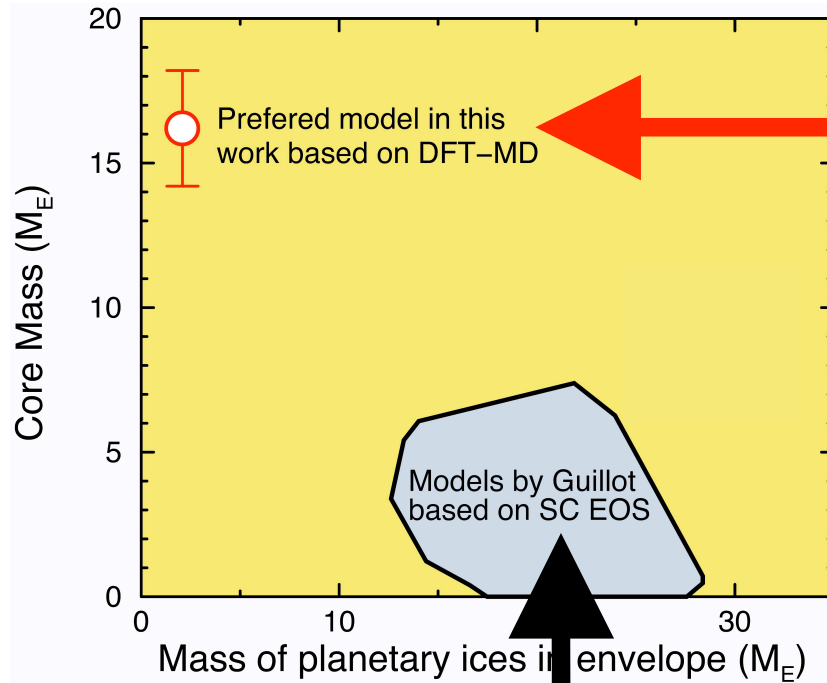
DFT-MD Simulations Predict A Massive Core in Jupiter



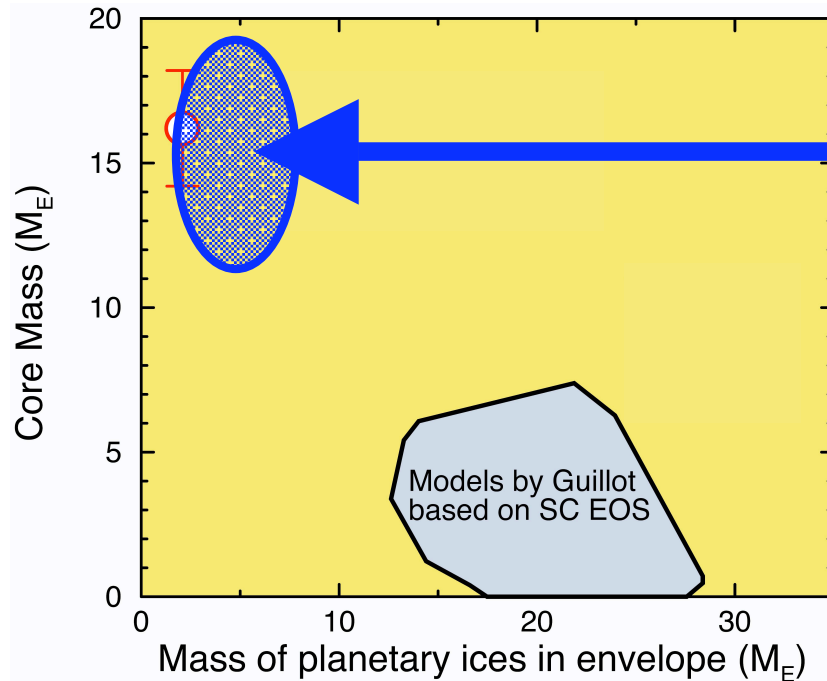
Core mass of $16 M_E$ predicted!



Our new 2-layer Jupiter model



Jupiter and Saturn made by same formation mechanism?



Saturn:

All models predict as **large core** of 10-20 Earth masses

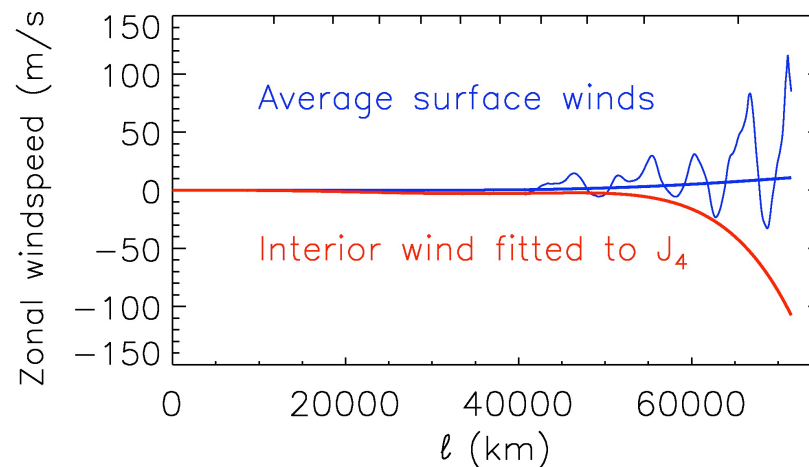
Small amount of ices in envelope
2-8 Earth masses

Guillot (2004)

Nasa mission to Jupiter: Juno, a low periapse orbiter
High quality gravity and magnetic field data expected during 2016.

Predictions: Jupiter does not rotate as a solid body \Rightarrow deep interior winds

Model	Equatorial radius (km)	$J_2 \times 10^6$	$J_4 \times 10^6$	$J_6 \times 10^6$
Observed	71492	14696.43 \pm 0.21	-587.14 \pm 1.68	34.25 \pm 5.22
Solid-body rotation	Matched	Matched	-620	37.5
Preferred model: deep winds	Matched	Matched	Matched	23.9

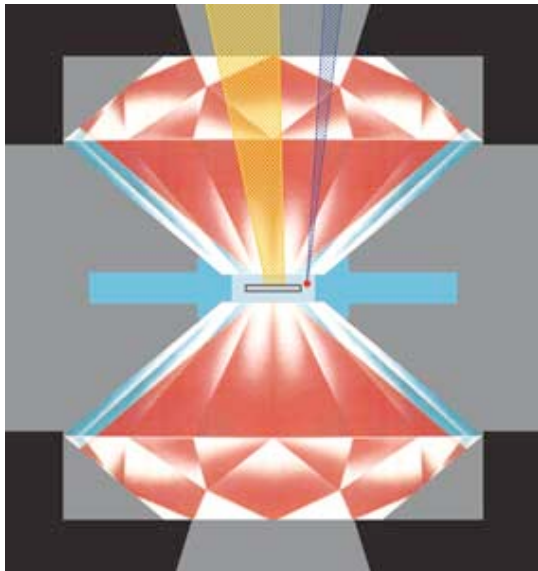


Summary: New Jupiter Model

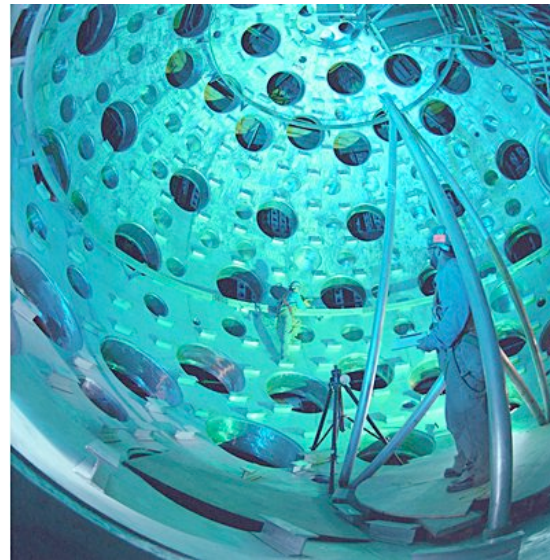
- EOS from **first-principles** simulations (DFT-MD)
- **Continuous molecular-to-metallic transition**
⇒ the planet is fully convective
- **Massive core** of 16 Earth masses predicted
- Small amount of ices in envelope
- Favored formation mechanism: **core-accretion**
- To match J_4 , we propose **differential rotation**

New Experimental Technique: Combination of **Static** and **Dynamic** Compression

1) Static compression
Diamond anvil cell

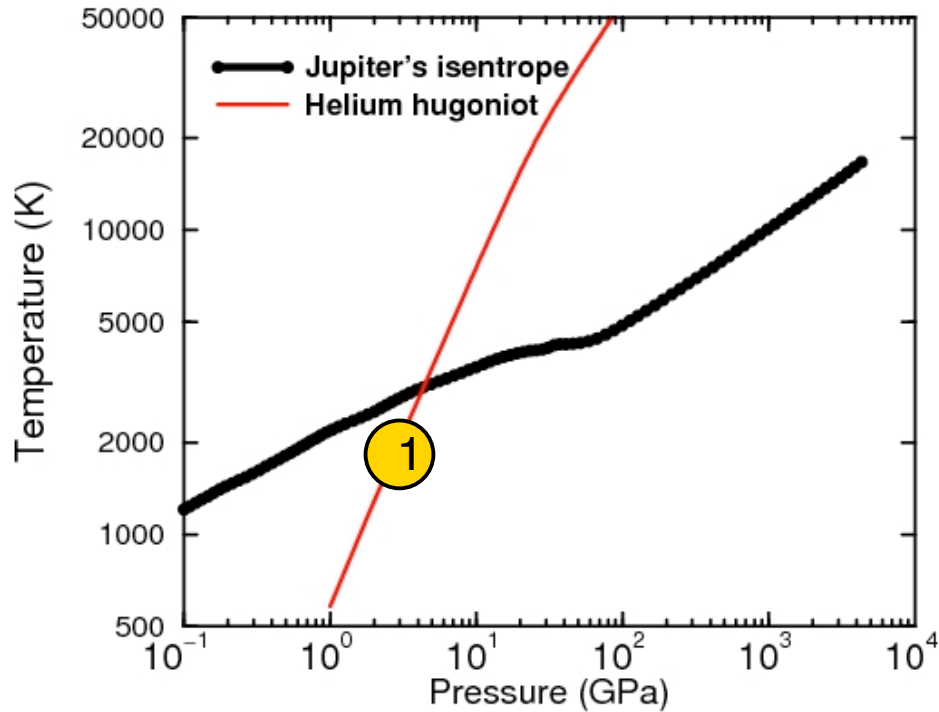


2) Dynamic shock comp.
Laser shocks

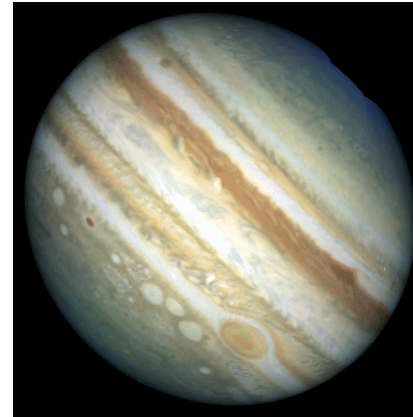


- LLNL-CEA collaboration
- Samples are **precompressed** in modified diamond anvil cell
- Precompression up to 1.5 GPa = 15 kbar

How far into in Jupiter's interior can be probed with precompressed shocks?

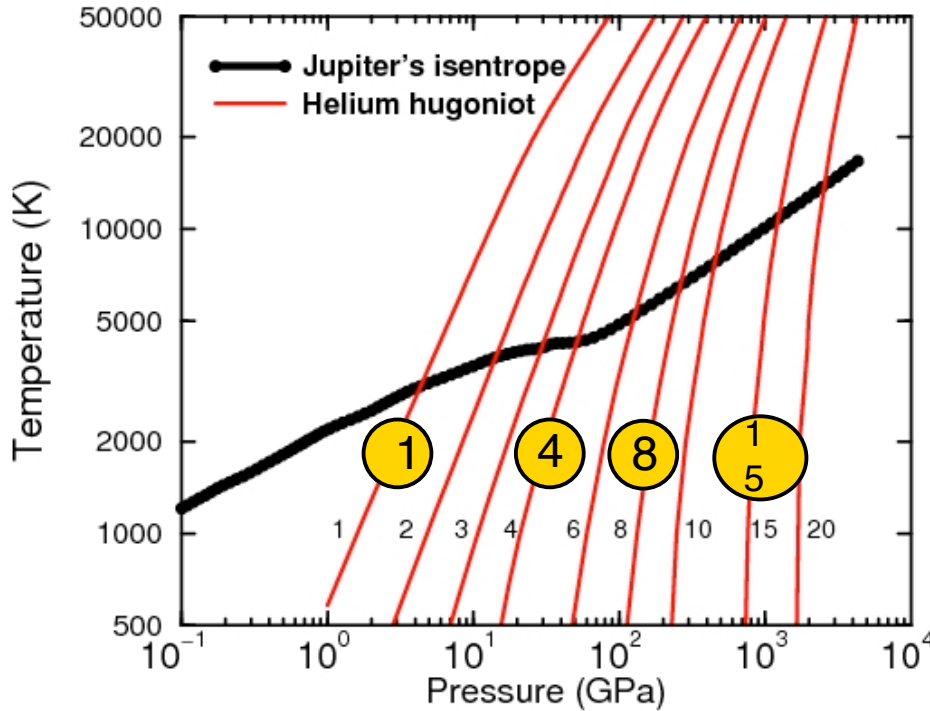


Where does the **helium** hugoniot intersect with Jupiter's isentrope?



	Precompression	P_0	$P_H(\text{GPa})$	$T_H(\text{K})$	Mass fraction	$1-R_H/R_J$
He	1	1 bar	4.4	3000	0.5%	3%

Precompression up to 100 GPa is needed to study Jupiter's interior

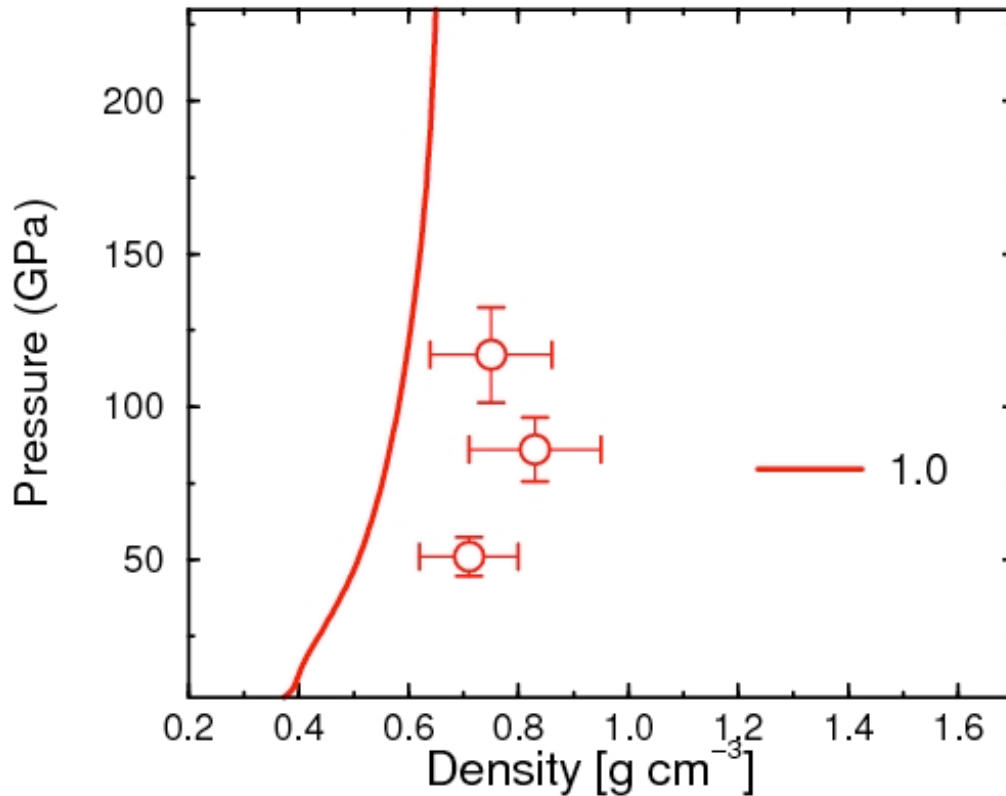


APS proceedings article (2007)

Militzer, Hubbard,
[arXiv:0707.4649](https://arxiv.org/abs/0707.4649)

	Precompression	P_0	P_H (GPa)	T_H (K)	Mass fraction	$1-R_H/R_J$
He	①	1 bar	4.4	3000	0.5%	3%
He	④	0.75 GPa	51	4200	5%	10%
He	⑧	13 GPa	261	6500	21%	
He	① ⑤	93 GPa	1200	11000	62%	

Comparison of PIMC Simulations with Laser Shock Experiments on Helium



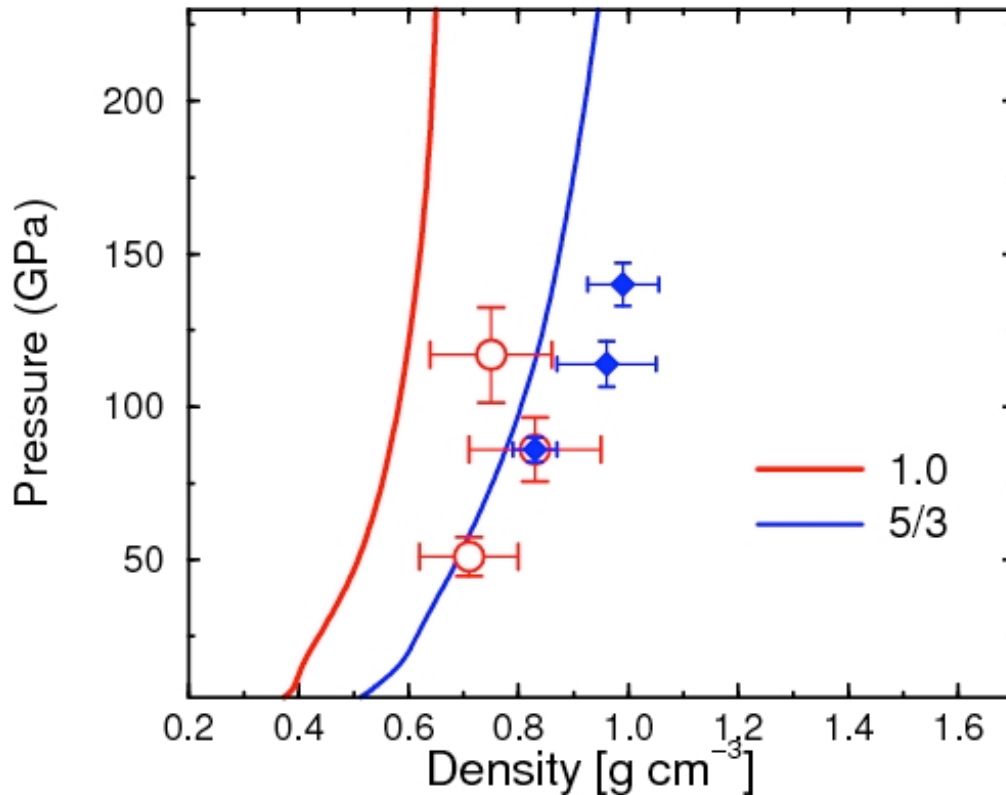
Principal Hugoniot:

PIMC predicts a lower compressibility.

Theory: Militzer, *Physical Review Letters*, **97** (2006) 175501;

Exp: Eggert *et al.* *Physical Review Letters*, 100 (2008) 124503.

Comparison of PIMC Simulations with Laser Shock Experiments on Helium



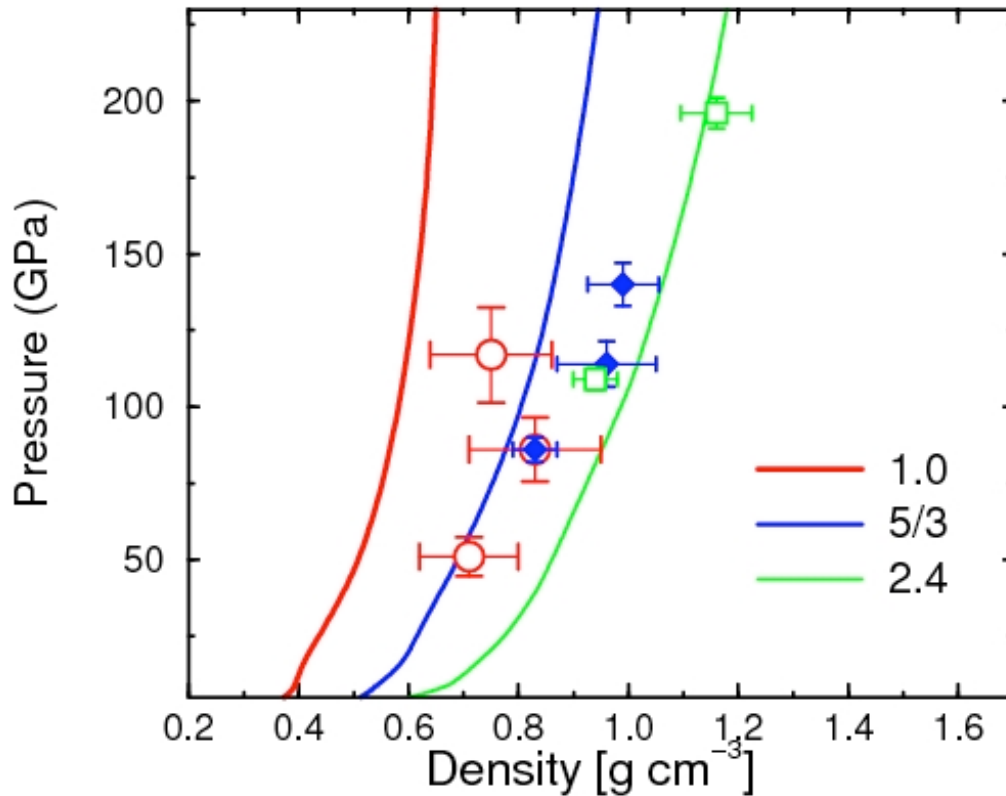
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Comparison of PIMC Simulations with Laser Shock Experiments on Helium

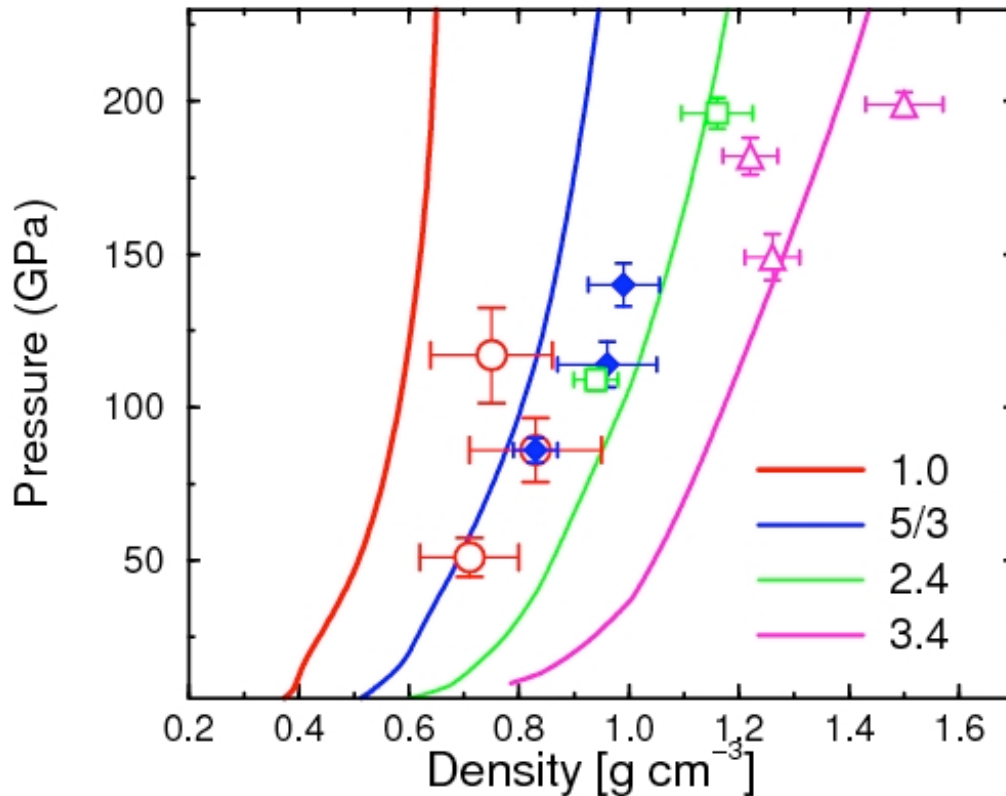


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Comparison of PIMC Simulations with Laser Shock Experiments on Helium



Principal Hugoniot:

PIMC predicts a lower compressibility

With 3...4-fold precompression:

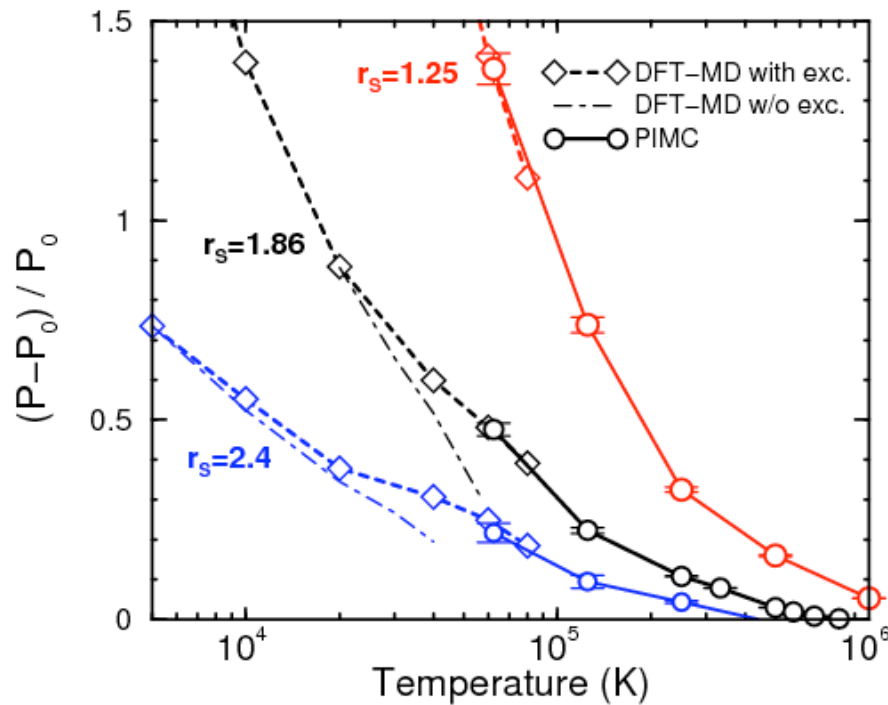
PIMC and experiment agree.

Theory: Militzer, *Physical Review Letters*, **97** (2006) 175501;

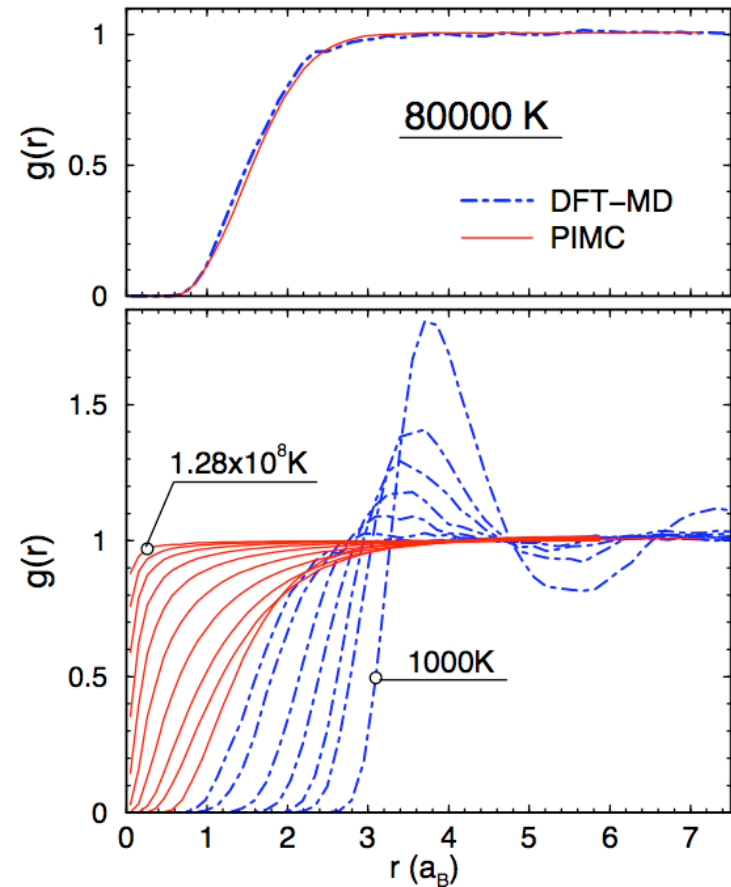
Exp: Eggert *et al.* *Physical Review Letters*, 100 (2008) 124503.

Marry **PIMC** and **DFT-MD** EOS Calculations for Dense Fluid Helium

Compare $P(T)$ for a wide range of densities:



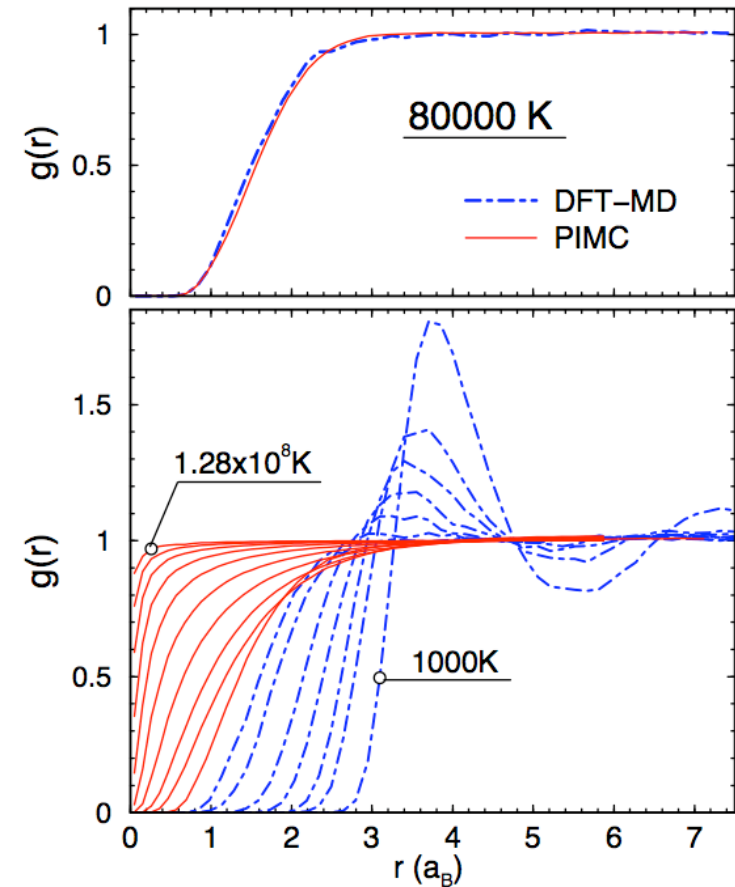
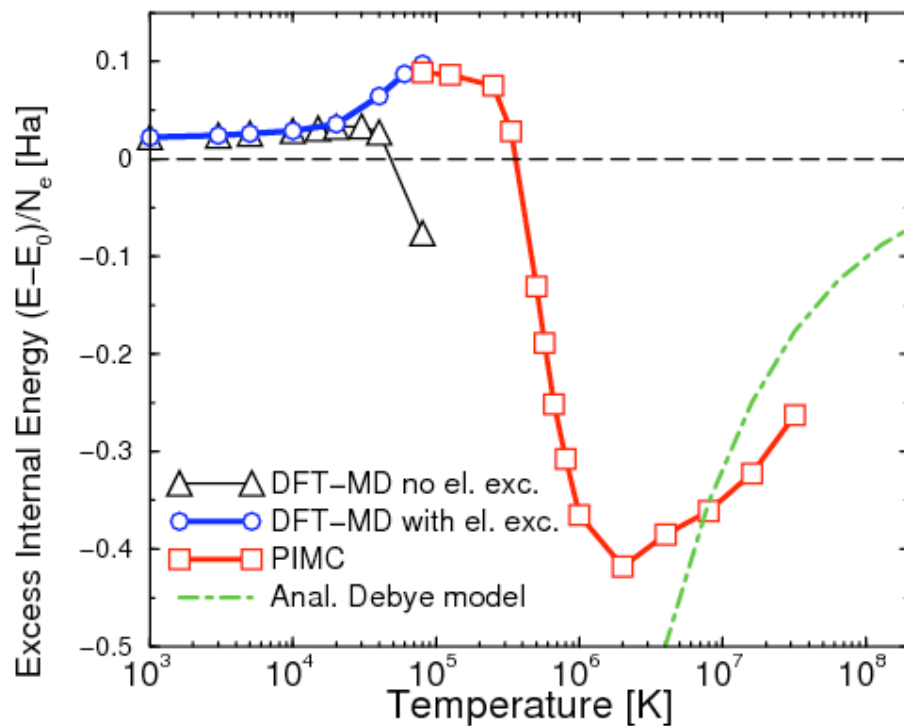
Good agreement found if excited states are included in DFT-MD.



B. Militzer, submitted to *Physical Review B* (2008), see [arXiv:0805.0317](https://arxiv.org/abs/0805.0317)

Marry **PIMC** and **DFT-MD** EOS Calculations for Dense Fluid Helium

Compare $E(T)$ for density of $r_s=1.86$



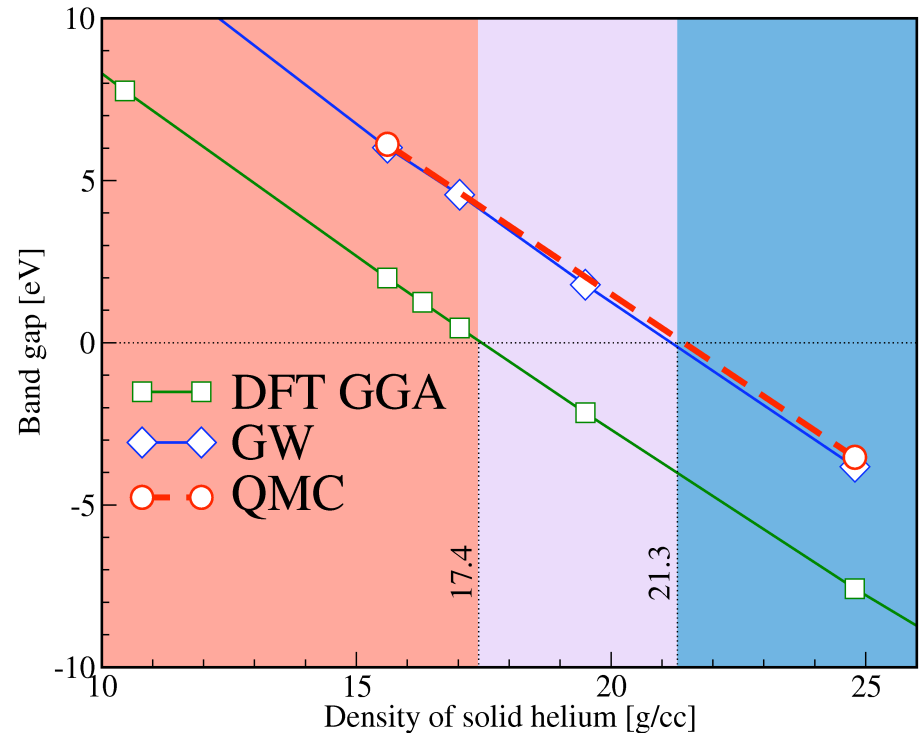
B. Militzer, submitted to *Physical Review B* (2008), see [arXiv:0805.0317](https://arxiv.org/abs/0805.0317)

QMC Calculation of the Metallization of Solid Helium under Pressure

Method comparison

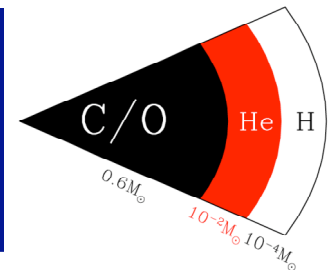
- ◆ QMC and GW: agreement
- ◆ GGA:
 - × Underestimates gap by 4eV
 - × 40% difference in pressure
 - × 20% difference in density

→ QMC done with **Casino (Cambridge)**.



White dwarf layers:

Solid helium metallizes at extreme pressure of 25.7 TPa. This transition is important for the heat transfer in hydrogen poor white dwarfs. Our article: [arXiv:0805.4433](https://arxiv.org/abs/0805.4433)



Summary and Job Advertisement

- **Theory:** First-principles simulations for giant planet interiors
 - 1) **Massive core of 16 Earth masses predicted**
 - 2) **Favored formation mechanism: core-accretion**
 - 3) **To match J_4 , we propose differential rotation**
- **Observations:** Rapidly expanding set of known extrasolar planets
- **Experiments:** New laser shock experiments find helium more than 5-fold compressible

- **Possibility to do a PhD in my group**
- **Post-doc position open in Computational Earth and Planetary Science**

<http://militzer.berkeley.edu>