

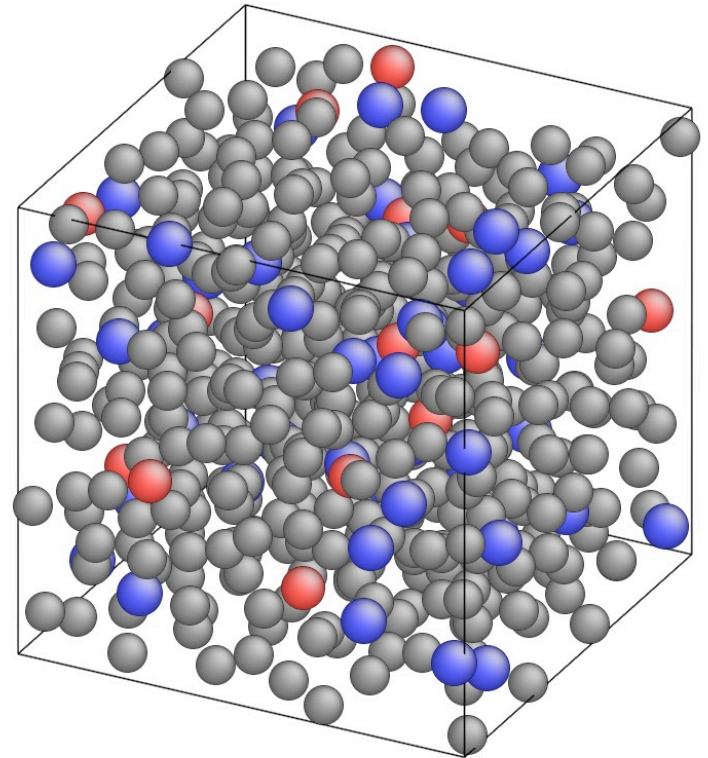
# Ab-Initio Molecular Dynamics Modeling of Molten Superalloys

**Mark Asta**

*Department of Chemical  
Engineering & Materials Science*



*ES09 Workshop  
University of California, Davis  
June 25, 2009*



# Acknowledgements

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**Stefano Angioletti-Uberti<sup>1</sup>, Mike Finnis<sup>1</sup>,  
Peter Lee<sup>1</sup>, James Lill<sup>2</sup>, Dallas Trinkle<sup>3</sup>, Chris Woodward<sup>4</sup>**

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*<sup>3</sup>Department of Materials Science and Engineering, University of Illinois Urbana-Champaign*

*<sup>4</sup>Materials and Manufacturing Directorate, Air Force Research Laboratory, Wright-Patterson Air Force Base*

## **Research Support**

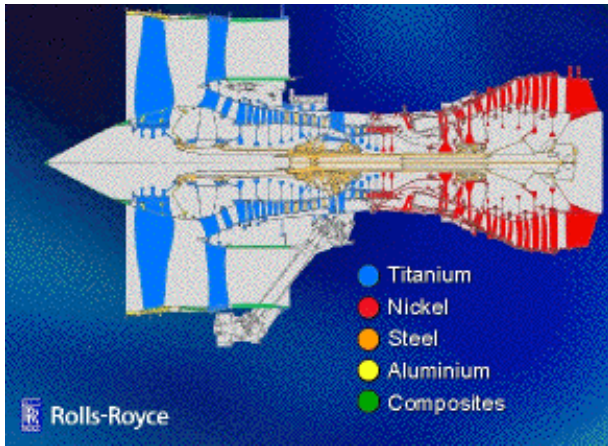
*US DOE (Office of Basic Energy Sciences)*

*AFOSR (Materials Engineering for Affordable New Systems Program)*

## **Supercomputing Resources**

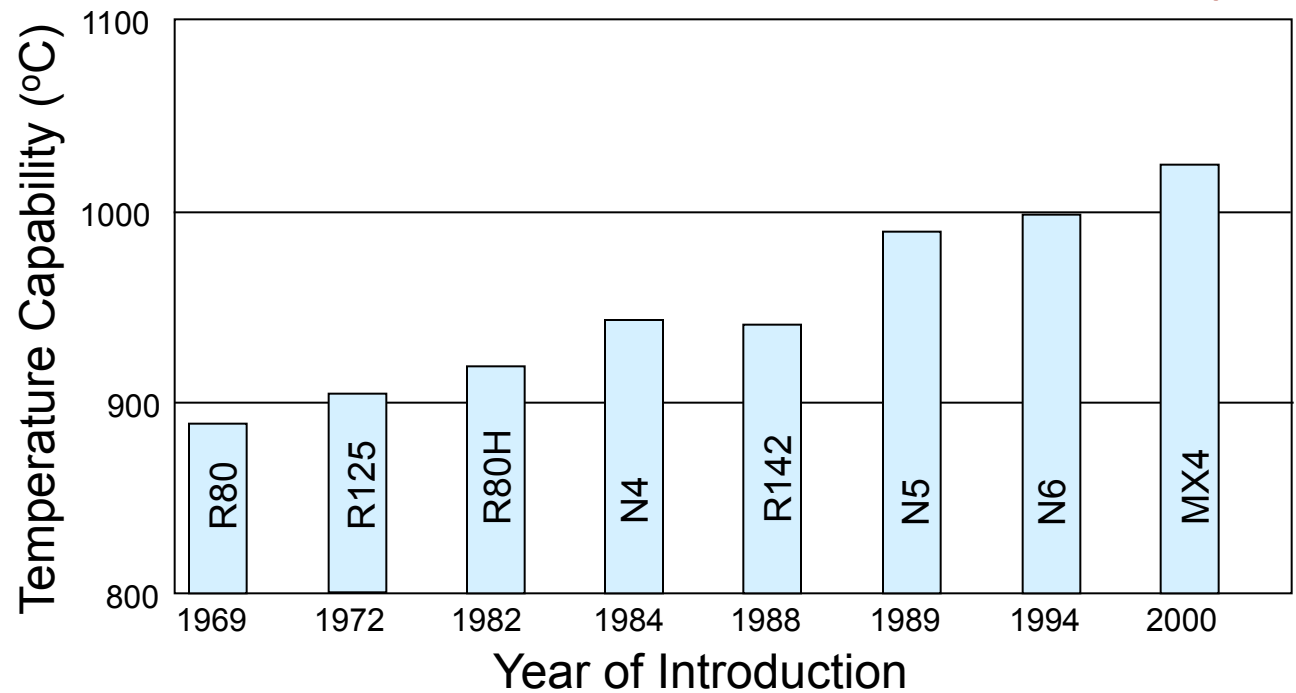
*DoD ASC-MSRC Challenge Grant*

# Ni-Based Superalloys



*Higher-T Materials → Higher Efficiency*

## Evolution of Temperature Capability



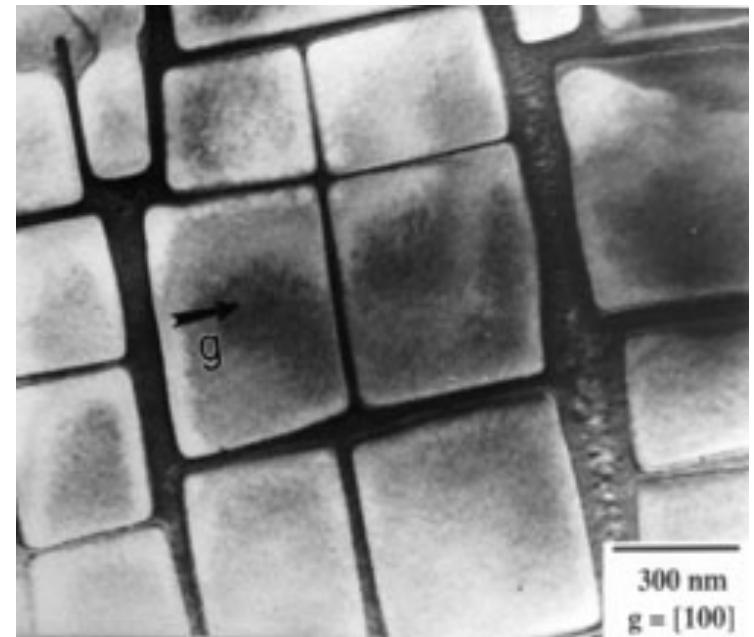
# 1st-Principles Thermo & Kinetic Modeling

- **Design of High-Temperature Materials**

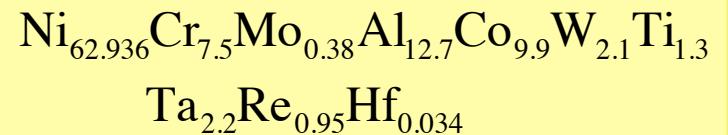
- Requires Data for Bulk and Interfacial Free Energies, Diffusion Kinetics, Lattice Parameters, ...
- In Many Cases Data Difficult to Measure (e.g., Metastable Phases, High T)

- **Role of 1<sup>st</sup>-Principles Calculations**

- Predictive Framework for Computing Structural, Thermodynamic and Kinetic Properties
- Applicable to Both Stable and Metastable Phases, Bulk and Interfaces, High & Low T



*CMSX-4 Superalloy  
A.Dlouhy and G.Eggeler*



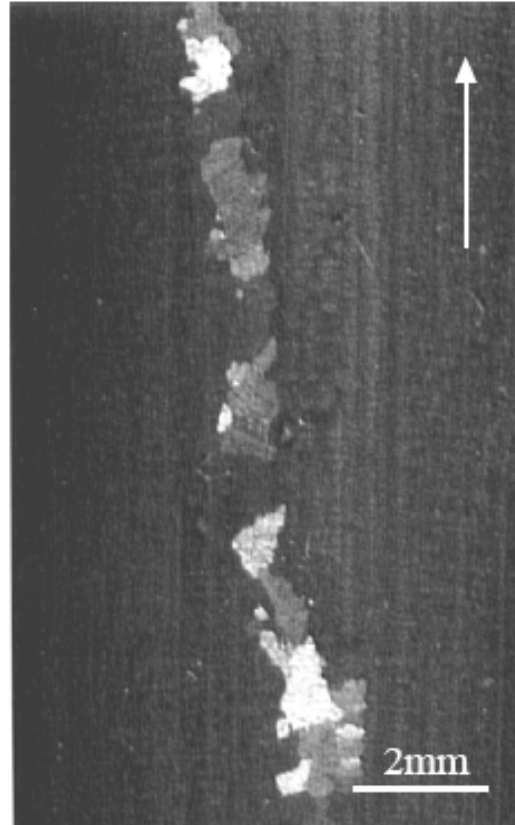
# Defects in Directional Solidification

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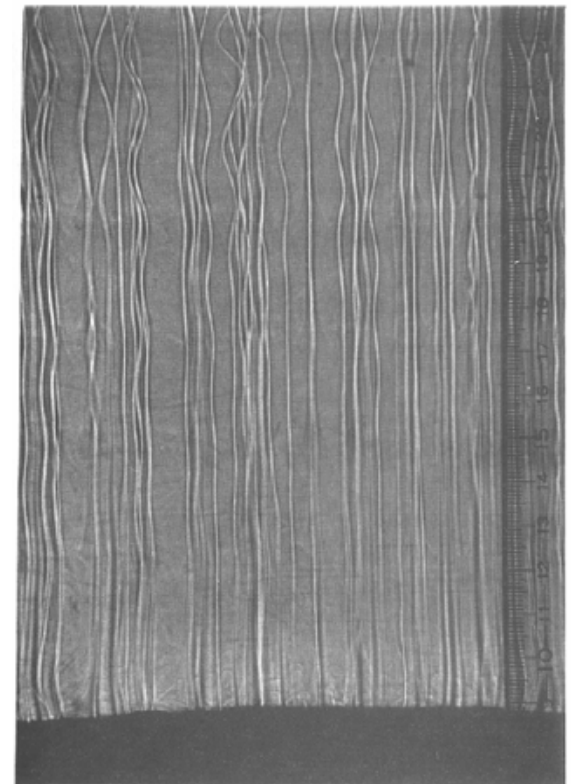
Directionally Solidified  
Superalloy Ingot  
(A. F. Giamei, UTRC)



Freckle Chain in Superalloy  
(Pollock and Murphy, 1996)



“Channels” in  $\text{NH}_4\text{Cl}$   
(Wooster, 1997)



# Freckle-Formation Prediction

Convective Instabilities Expected When  
Critical Value of  $Ra_s$  Exceeded  
(e.g., Beckermann et al., 2000)

$$Ra_s = \frac{(\Delta\rho / \rho_0) g K h}{\alpha \nu}$$

$Ra_s$  → Ratio of Buoyancy to Retarding Frictional Forces

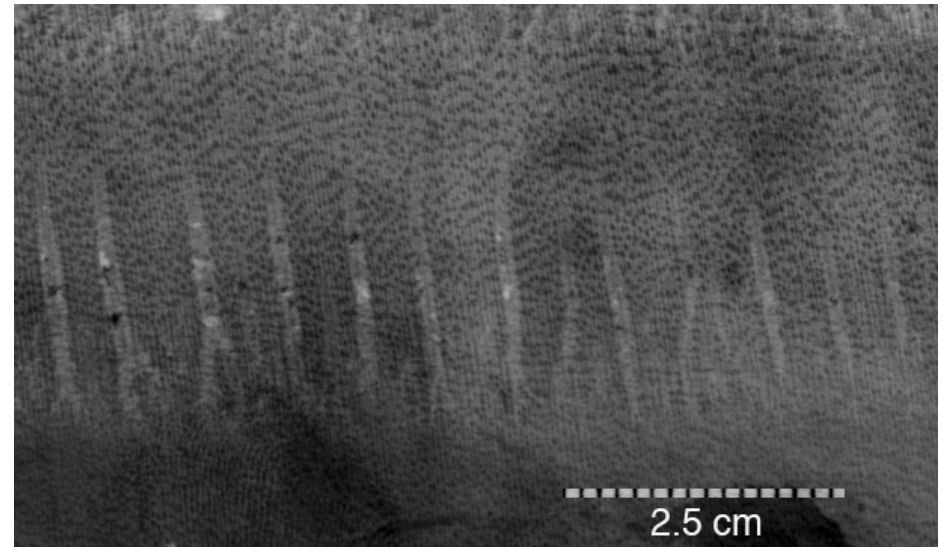
$\Delta\rho$  → Variation in Mass Density Across Mushy Zone

$g$  → Gravity

$K$  → Mushy Zone Permeability

$h$  → Mushy Zone Height

$\alpha, \nu$  → Thermal Diffusivity and Viscosity



Channels in a Superalloy  
(Courtesy T. Pollock)

# Mass Density

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$$\rho(\{x_\alpha\}, T) = \frac{1}{V(\{x_\alpha\}, T)} \sum_{\alpha} x_{\alpha} M_{\alpha}$$

$x_{\alpha}$  → Mole Fraction of Species  $\alpha$

$M_{\alpha}$  → Molar Mass of Species  $\alpha$

$V(\{x_{\alpha}\}, T)$  → Molar Volume

*Accurate Parametrization of **Temperature and Composition**  
Dependent **Molar Volume** and **Solute Partitioning** Required to  
Predict Density Inversions Across Mushy Zone*

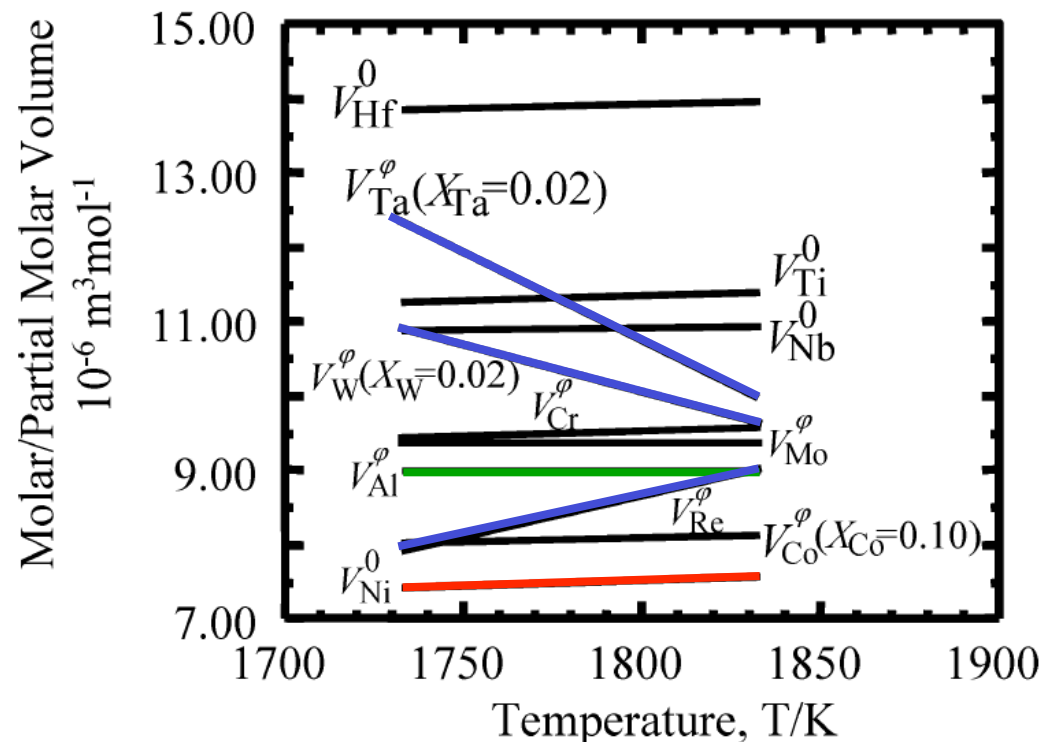
# Molar-Volume Parametrization

K. Mukai, Z. Li and K. C. Mills, *Met. Trans. B*, vol. **36B**, 255 (2005)

$$V(\{x_\alpha\}, T) \equiv \sum_\alpha x_\alpha \bar{V}_\alpha(x_\alpha, T)$$

$\bar{V}_\alpha(\{x_\alpha\}, T) \rightarrow$  Partial Molar Volume of Species  $\alpha$

- Assume  $\bar{V}_\alpha$  Depends Only on  $x_\alpha$  and  $T$ 
  - Neglects Multicomponent Interactions
- Molar Volumes Fit to Experimentally Measured Binary Alloy Data
  - Large Variations of Partial Molar Volumes w/ Temperature & Composition Obtained from Fits





# Quantum MD for Molten Ni-Based Alloys

- **Simulation Details**

- (NVT) Dynamics at  $T=1750$  &  $1830$  K
- Time Step:  $0.002$  or  $0.003$  ps
- $\sim 5$ - $10$  ps Simulation Time
- 500 Atom System Size
- VASP Code on 64 Processors
- Ultrasoft Pseudopotentials and PAW

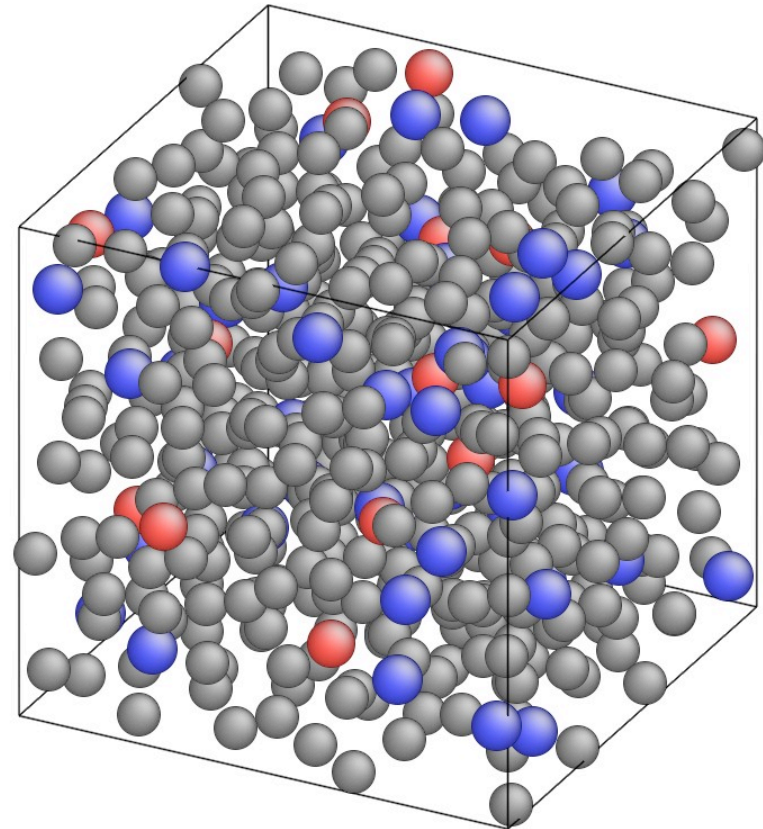
- **Volumes**

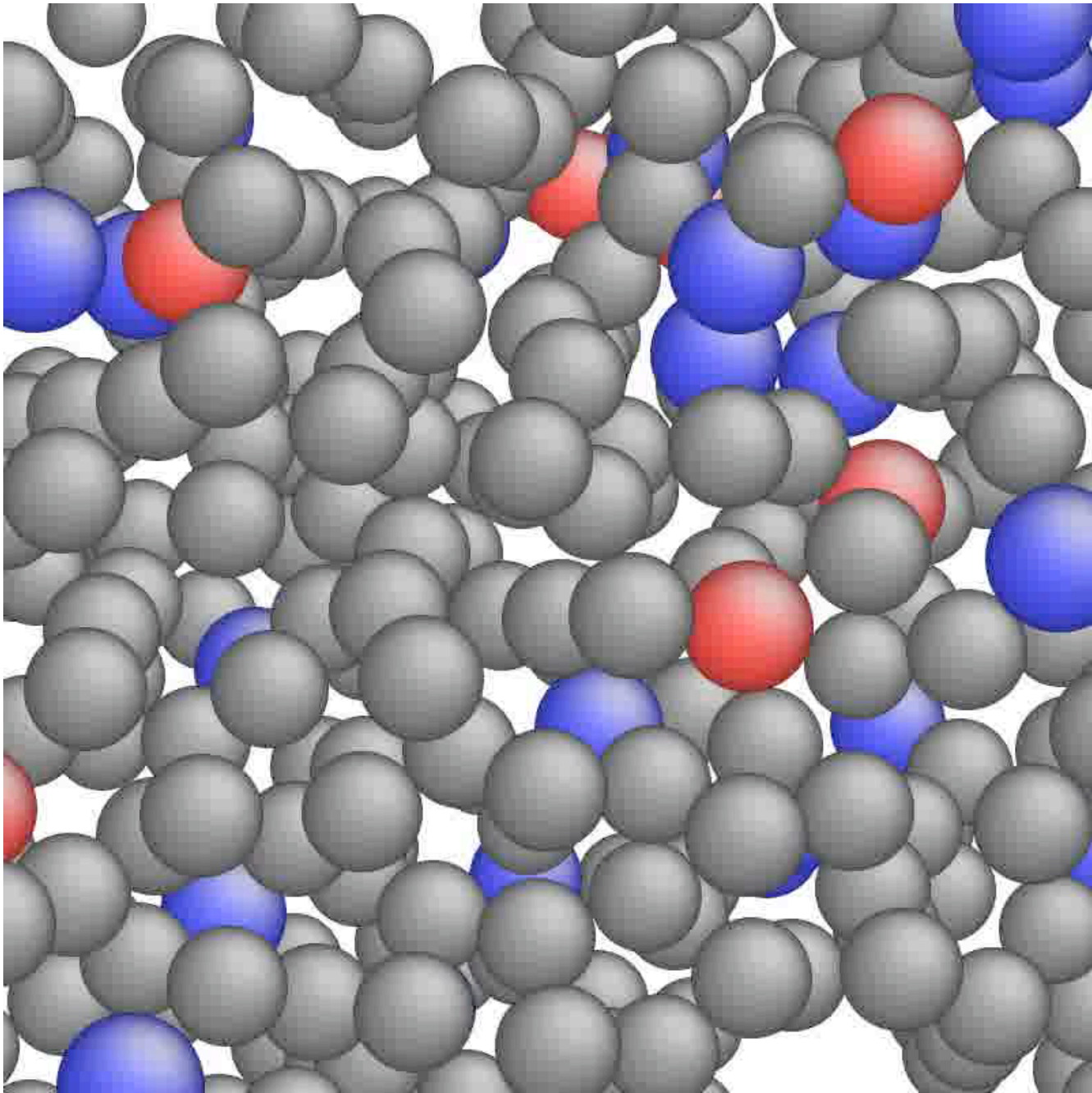
- 3 or 4 Volumes:  $0.95 V_{\text{ref}} - 1.05 V_{\text{ref}}$
- Equations of State and Equilibrium Volume

- **Systems**

- Elemental Ni and Al
- Binary Ni-Al, Ni-W, Ni-Re, Ni-Ta
- Ternary Ni-Al-W, Ni-Al-Re, Ni-Al-Ta

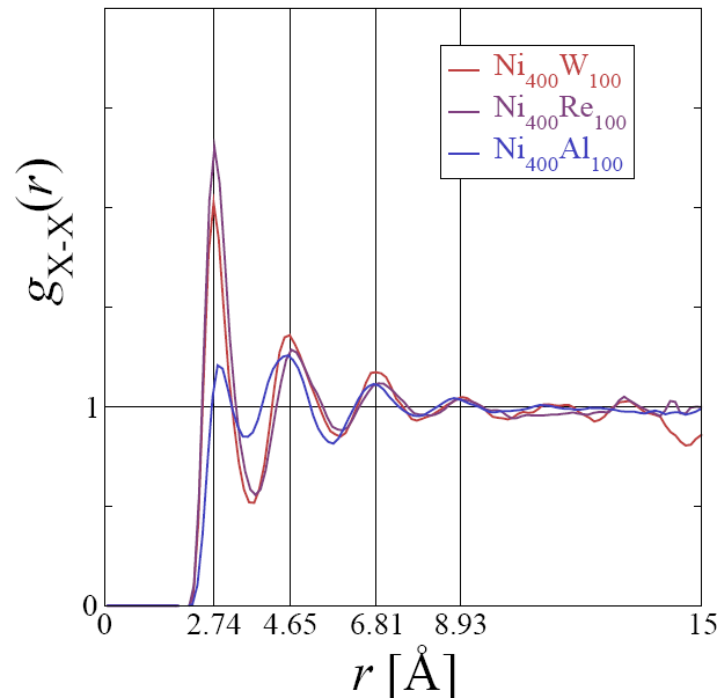
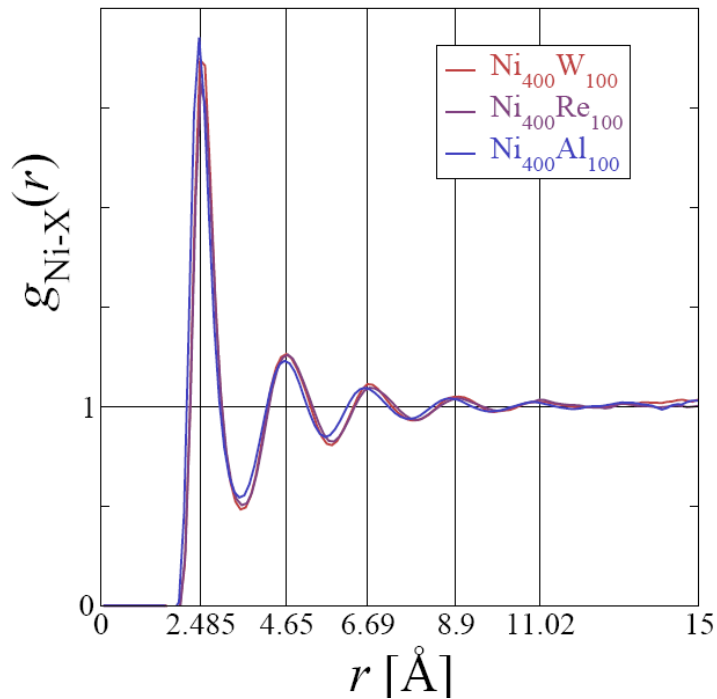
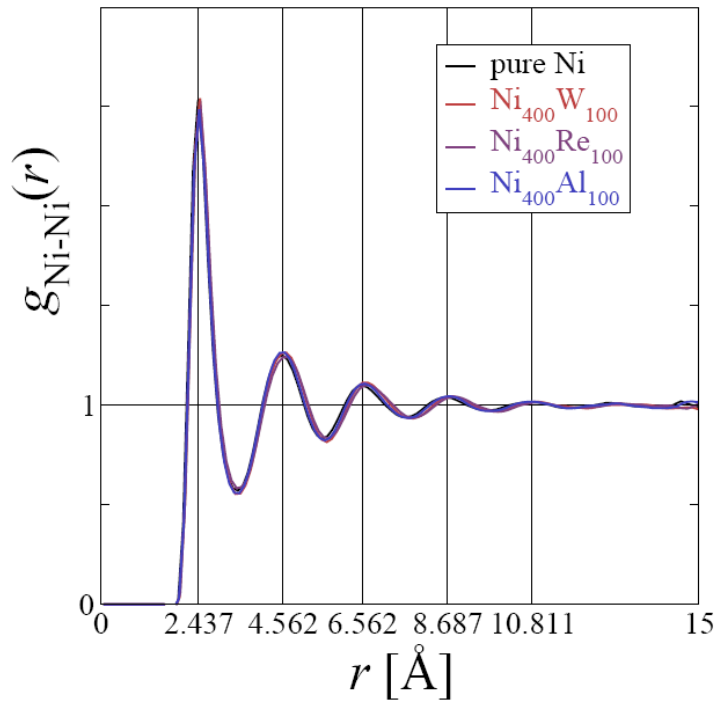
*Snapshot of Simulation for*





# Radial Distribution Functions

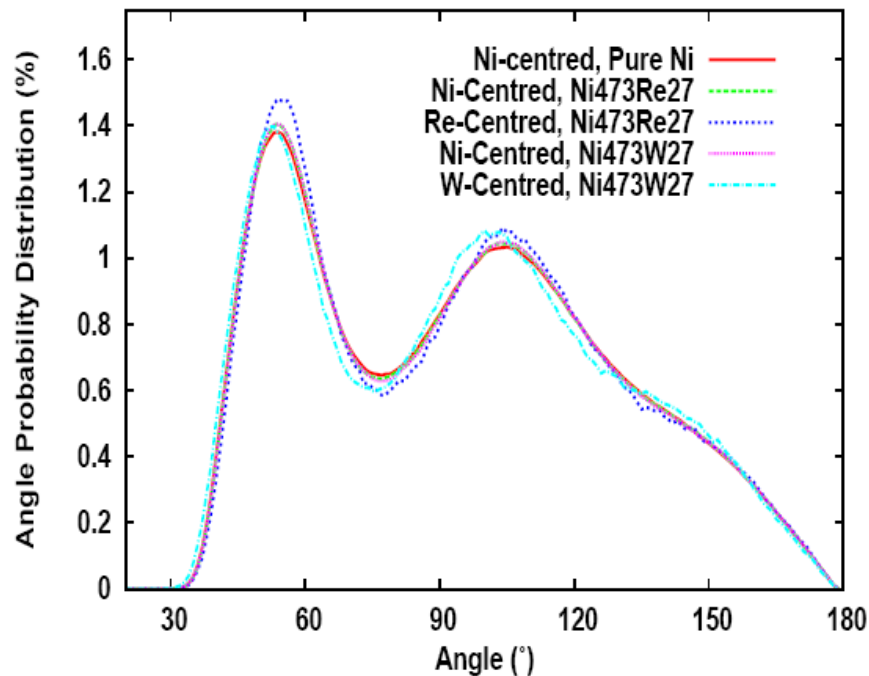
- Results Indicate Tendency for Chemical Short-Range Ordering
- Nearest-Neighbor Bond Lengths  
 $R_{NiX} < (R_{NiNi} + R_{XX})/2$
- Nearest-Neighbor Coordinations  
 $g_{NiX}(R_{NiX}) > g_{NiNi}(R_{NiNi}) > g_{XX}(R_{XX})$



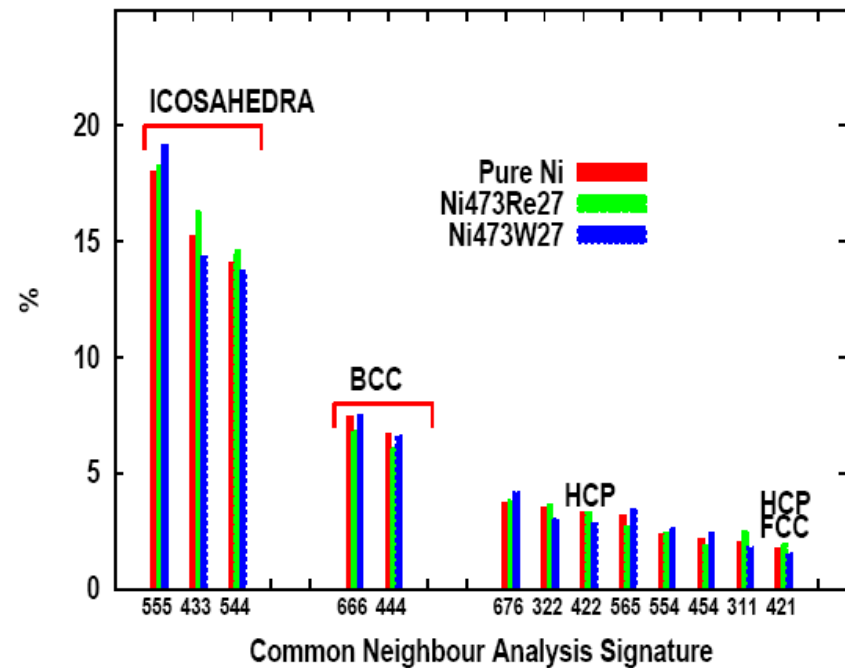
# Liquid Structure

*Ni, Ni<sub>473</sub>W<sub>27</sub> and Ni<sub>473</sub>Re<sub>27</sub> at 1830 K*

## Bond Angle Distributions

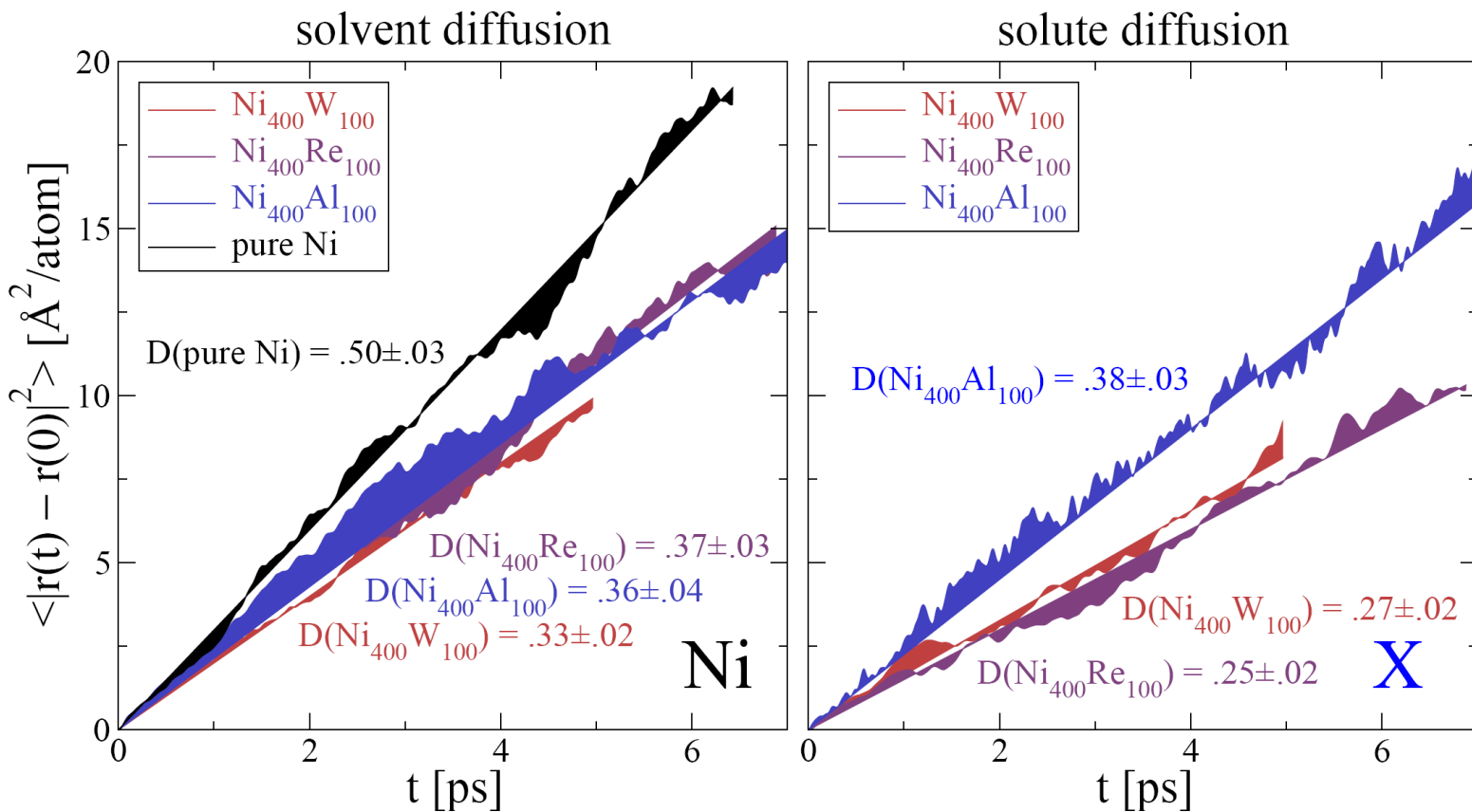


## Common-Neighbor Analysis



***Liquid Structures Display Short Range Order  
Featuring Predominantly Icosahedral and BCC Local Structures***

# Diffusion Constants



W Tracer Diffusion  
Constants

$$(2.4 \pm 0.2) \times 10^{-5} \text{ cm}^2/\text{s}$$

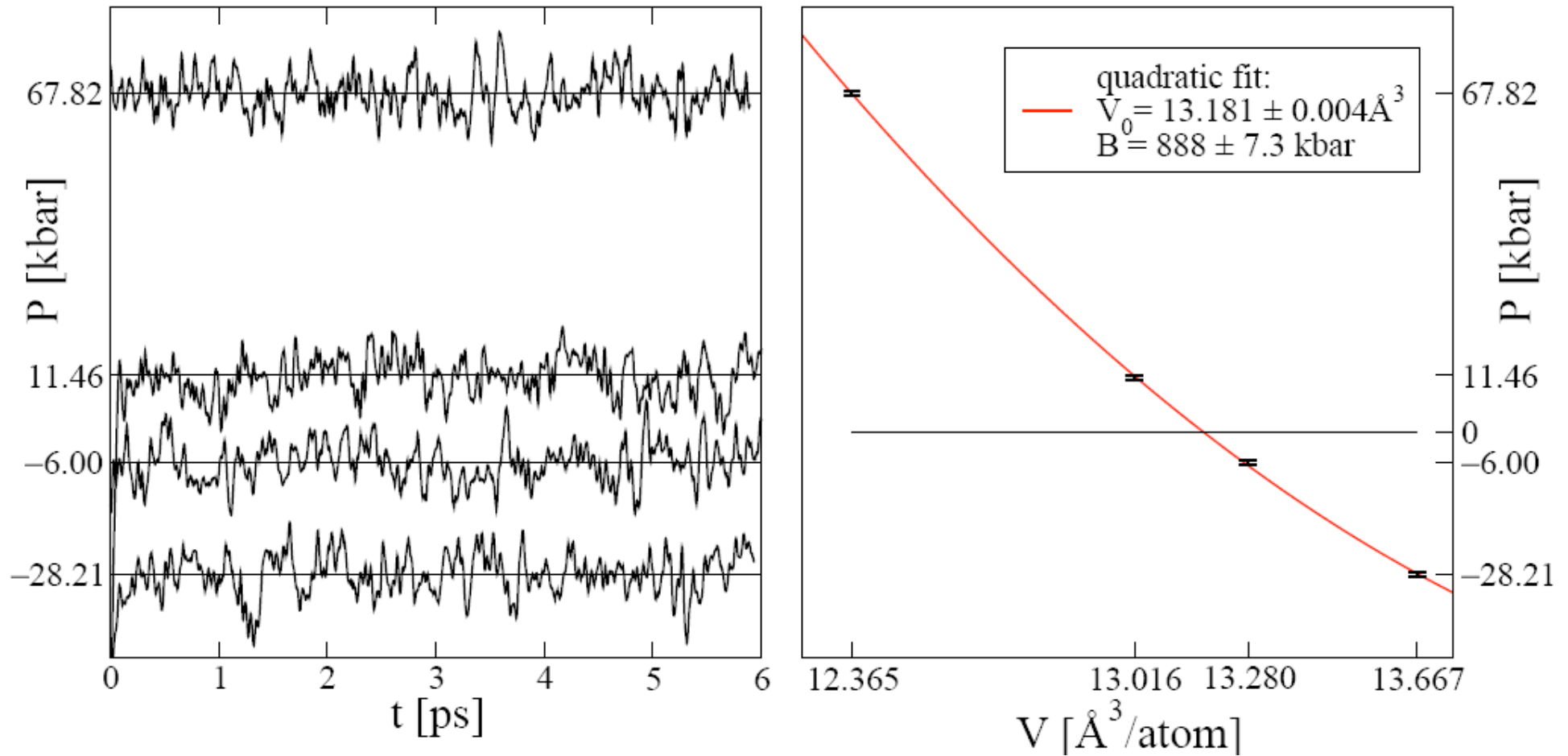
Ni-0.52 at.%W, 1755-2022 K  
Leonard et al. (2004)

$$(2.7 \pm 0.2) \times 10^{-5} \text{ cm}^2/\text{s}$$

Ni<sub>4</sub>W, 1830 K  
Current Work

# Pressure-Volume Relations

$Ni_{400}Al_{100}$  at  $T=1830$  K



# Molar Volumes for Molten Ni-Based Alloys

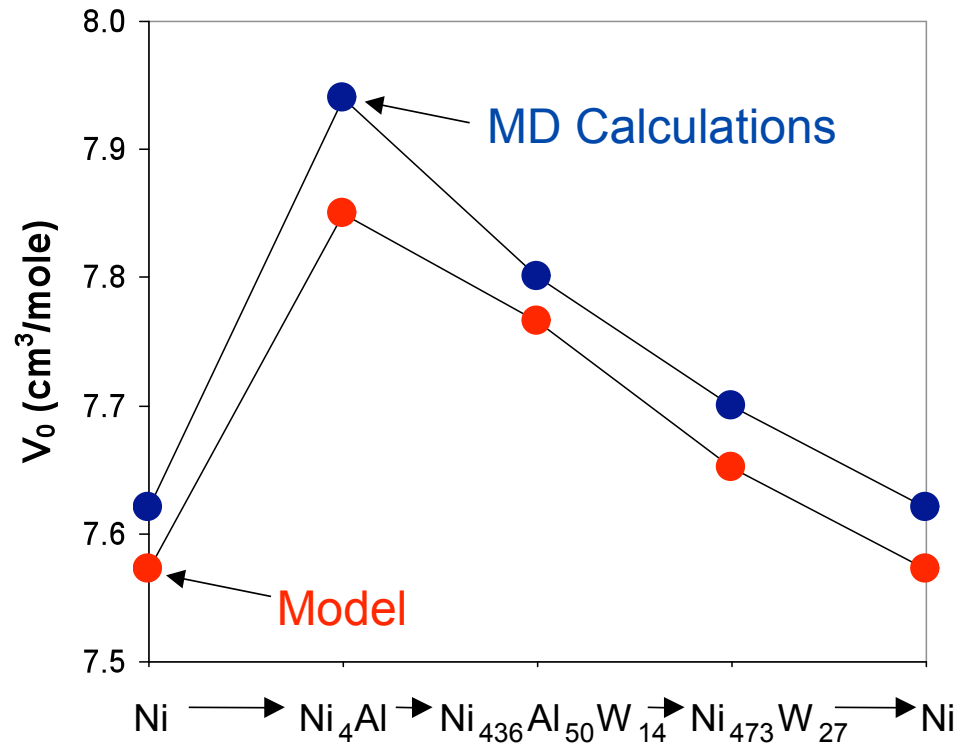
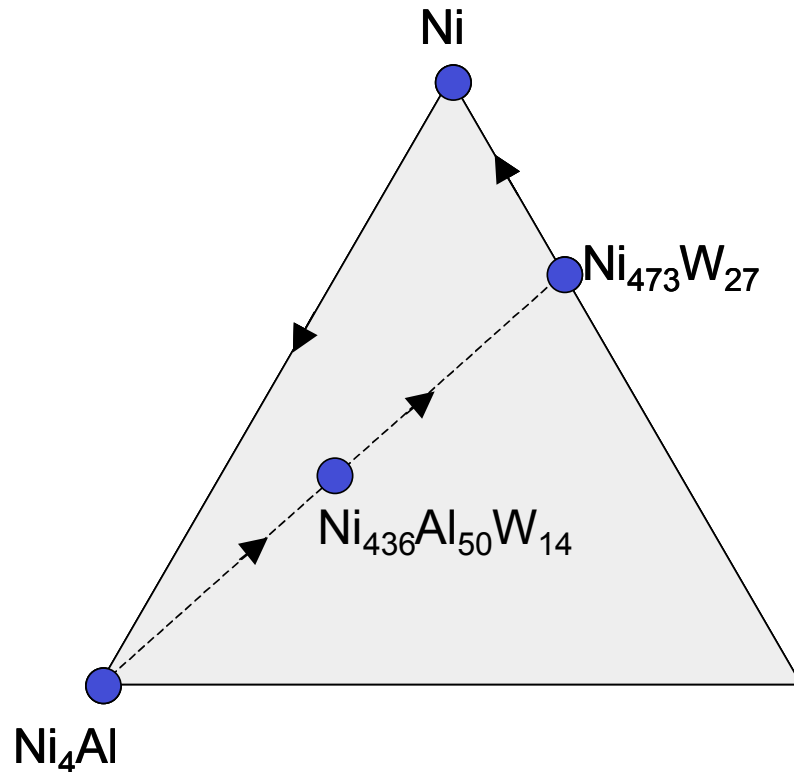
*MD Calculations, Mukai Model and Experiment*

*All Volumes in Units of cm<sup>3</sup>/mole*

Composition	T=1750 K			T=1830 K		
	Mukai	AIMD	$V_{\text{AIMD}}/V_{\text{EXPT}}$	Mukai	AIMD	$V_{\text{AIMD}}/V_{\text{EXPT}}$
Ni <sub>500</sub>	7.4597	7.57(1)	1.015	7.5724	7.62(1)	1.006
Ni <sub>400</sub> Al <sub>100</sub>	7.7628	7.88(1)	1.015	7.8492	7.94(1)	1.012
Ni <sub>473</sub> W <sub>27</sub>	7.5534	7.66(1)	1.014	7.6520	7.70(1)	1.006
Ni <sub>400</sub> W <sub>100</sub>	6.5192	7.94(1)		7.3294	7.98(1)	
Ni <sub>473</sub> Re <sub>27</sub>	7.4940	7.65(1)		7.6499	7.69(1)	
Ni <sub>400</sub> Re <sub>100</sub>	7.5869	7.91(1)		7.8593	7.93(1)	
Ni <sub>473</sub> Ta <sub>27</sub>	7.3310	7.70(1)		7.5053	7.75(1)	
Ni <sub>400</sub> Ta <sub>100</sub>	1.1043	8.14(1)		4.1440	8.18(1)	
Ni <sub>436</sub> Al <sub>50</sub> W <sub>14</sub>	7.6920	7.77(1)		7.7655	7.80(1)	
Ni <sub>436</sub> Al <sub>50</sub> Re <sub>14</sub>	7.6921			7.7510	7.80(1)	
Ni <sub>436</sub> Al <sub>50</sub> Ta <sub>14</sub>	7.6911	7.79(1)		7.7553	7.84(1)	

# Molar Volume Predictions

Results for Ternary Ni-Al-W Alloys at  $T=1830\text{ K}$



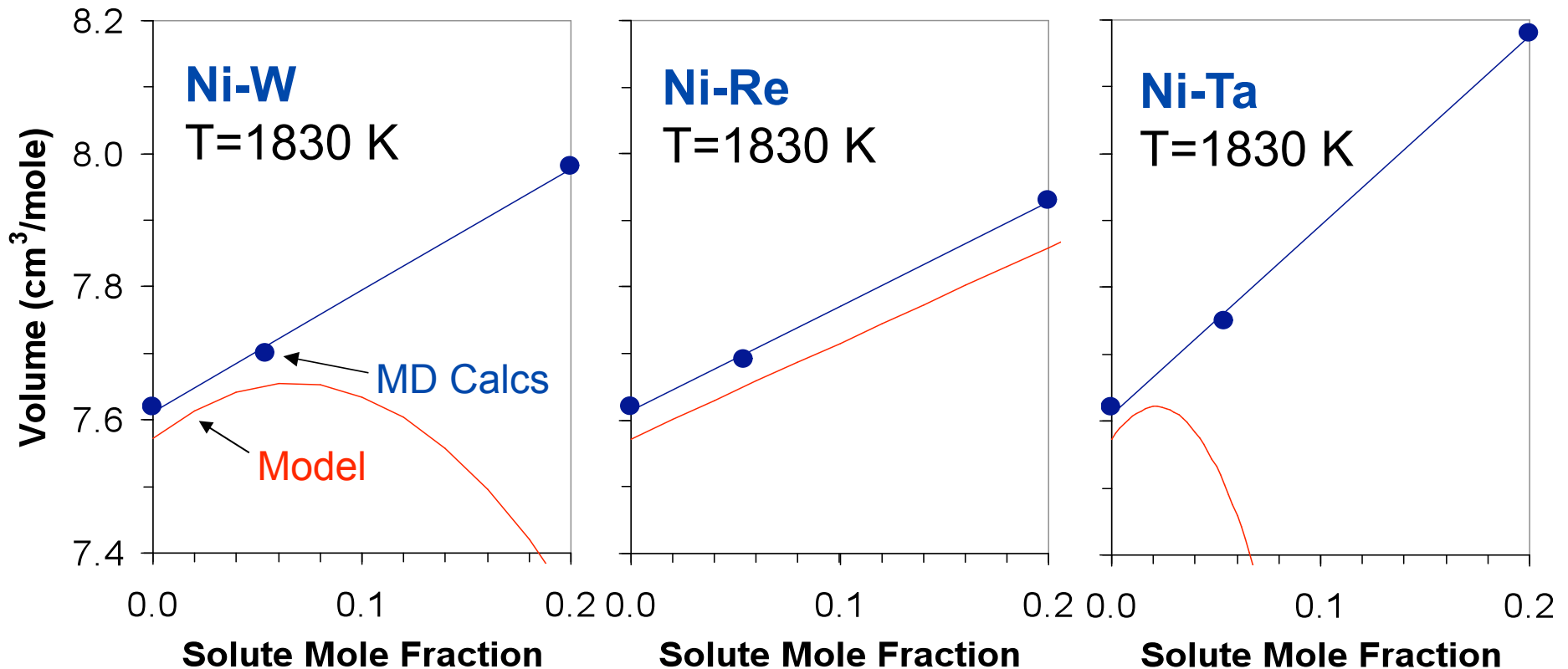
*Similar results obtained for Ni-Al-Re and Ni-Al-Ta*

*Results suggest high accuracy of published parameterization for compositions bounded by experimental measurements*



# Molar Volume Predictions

*Results for Binary Alloys*



*MD results demonstrate limitations in accuracy of published parameterizations when extrapolated beyond bounds of experimental measurements*

# Partial Molar Volumes

## Results for Infinite Dilution

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### T=1830 K

	Ni-W	Ni-Re	Ni-Ta
MD Calcs	$\bar{V}_W^\infty / \bar{V}_{Ni}^0 = 1.24$	$\bar{V}_{Re}^\infty / \bar{V}_{Ni}^0 = 1.20$	$\bar{V}_{Ta}^\infty / \bar{V}_{Ni}^0 = 1.37$
Model	$\bar{V}_W^\infty / \bar{V}_{Ni}^0 = 1.33$	$\bar{V}_{Re}^\infty / \bar{V}_{Ni}^0 = 1.19$	$\bar{V}_{Ta}^\infty / \bar{V}_{Ni}^0 = 1.61$

### T=1750 K

	Ni-W	Ni-Re	Ni-Ta
MD Calcs	$\bar{V}_W^\infty / \bar{V}_{Ni}^0 = 1.25$	$\bar{V}_{Re}^\infty / \bar{V}_{Ni}^0 = 1.22$	$\bar{V}_{Ta}^\infty / \bar{V}_{Ni}^0 = 1.38$
Model	$\bar{V}_W^\infty / \bar{V}_{Ni}^0 = 1.55$	$\bar{V}_{Re}^\infty / \bar{V}_{Ni}^0 = 1.08$	$\bar{V}_{Ta}^\infty / \bar{V}_{Ni}^0 = 2.13$

*MD results suggest relatively weak temperature dependencies for partial molar volumes*

# Ab-Initio Calculations of Solid-Liquid Phase Equilibria

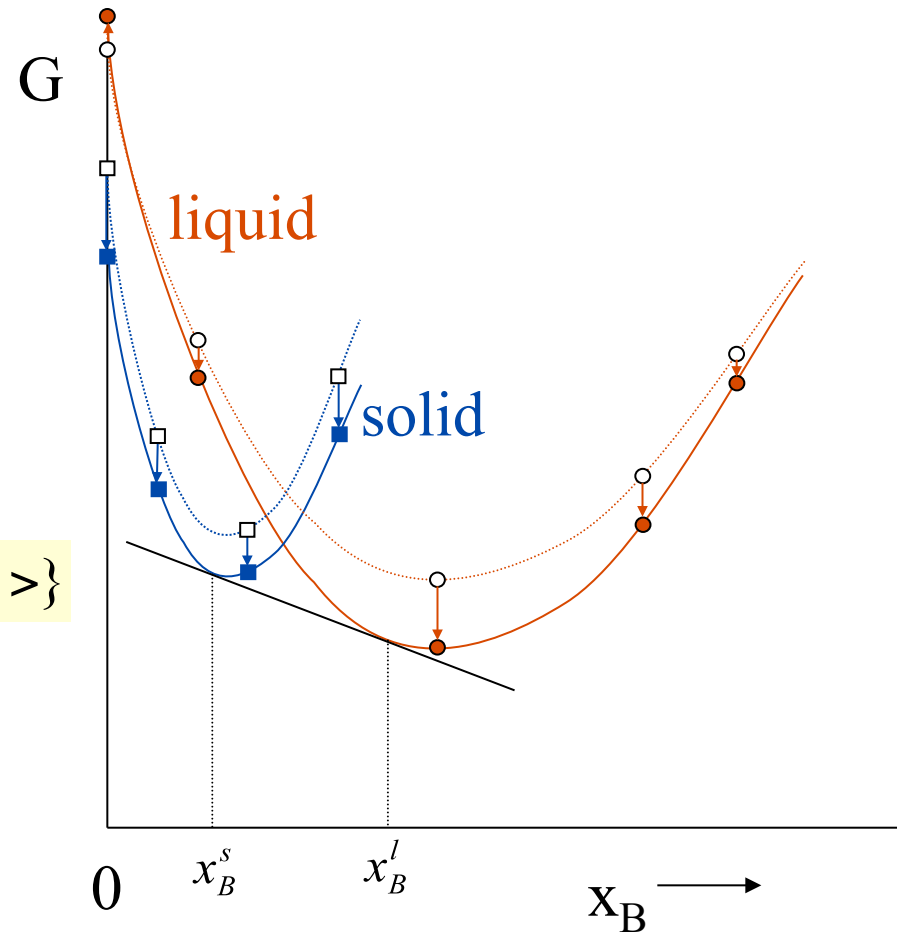
## Free Energies from “Perturbation” Approach

- **Free Energies Readily Derived for Classical Interatomic Potentials**
  - *Thermodynamic Integration Employing MD and Monte-Carlo Methods*
- **“Corrections” to Classical Free Energies to Achieve DFT Accuracy**

$$F_B - F_A = -k_B T \ln \{ \langle \exp[-(U_B - U_A)/k_B T] \rangle \}$$

$\langle \dots \rangle$  Ensemble Average over States of Reference System A

$U_B - U_A$  Energy Difference Between System B and A for Given State of System A



S. Angioletti-Uberti, M. Asta, M. W. Finnis and P. D. Lee, *Phys. Rev. B* (2008)

**Closely related:** C. W. Greeff, *J. Chem. Phys.* (2008)

# Convergence Properties

## *Description of Test System*

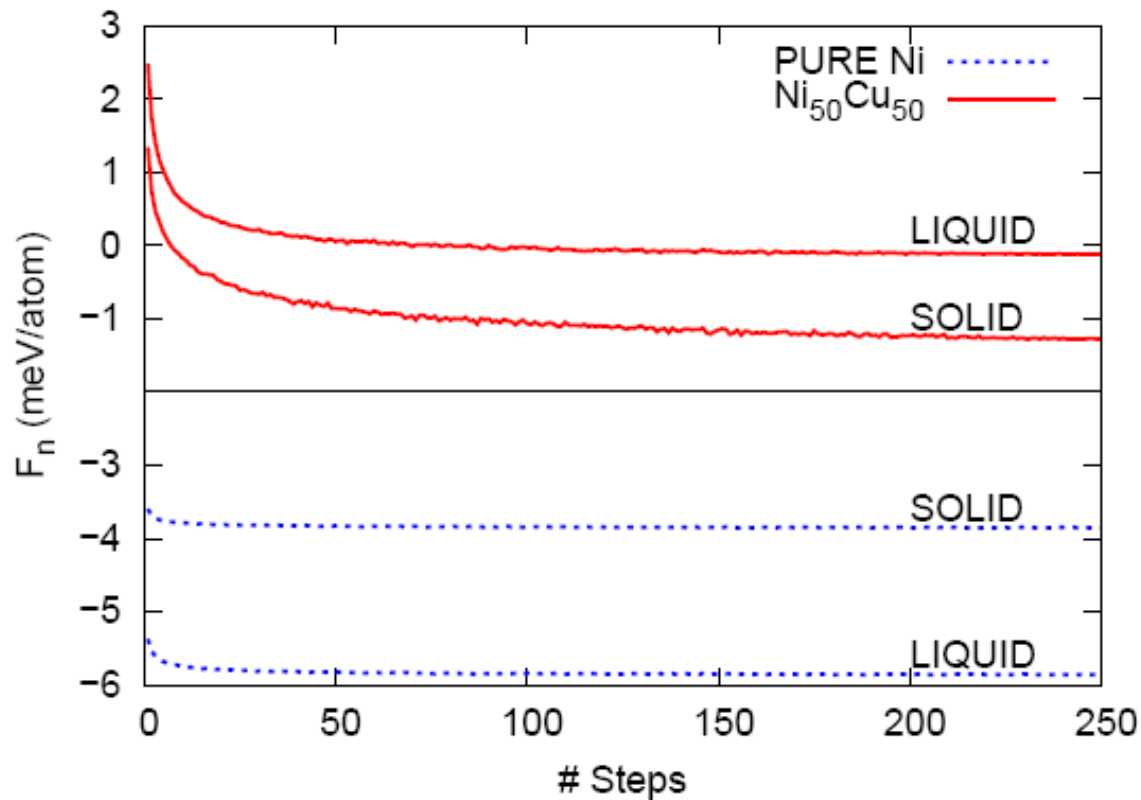
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- Two classical Embedded Atom potentials for Ni-Cu
  - **Reference:** “smf7” potential due to Foiles (1985)
  - **Target:** “u3” potential due to Foiles, Baskes & Daw (1984)
- Melting temperature and solidus/liquidus boundaries
  - Known to be approximately 100 K higher for smf7 than u3 from previous thermodynamic integration calculations
- Compositions and trajectories
  - **Solid and Liquid Pure Ni:** Reference trajectory from NVT MD
  - **Solid and Liquid NiCu Equiatomic Alloy:** Reference trajectory from NVT Monte-Carlo
- Calculated quantities
  - Free Energy Differences
  - Differences in Pure-Ni Melting Temperatures

# Convergence Properties

## Results for Test System

$$F_B - F_A \approx F_n \equiv -k_B T \ln \left[ \frac{1}{n} \sum_{i=1}^n \exp(-[U_B(\sigma_i^A) - U_A(\sigma_i^A)]/k_B T) \right]$$



Convergence to within 1 meV/atom

~20 Steps for Pure Element

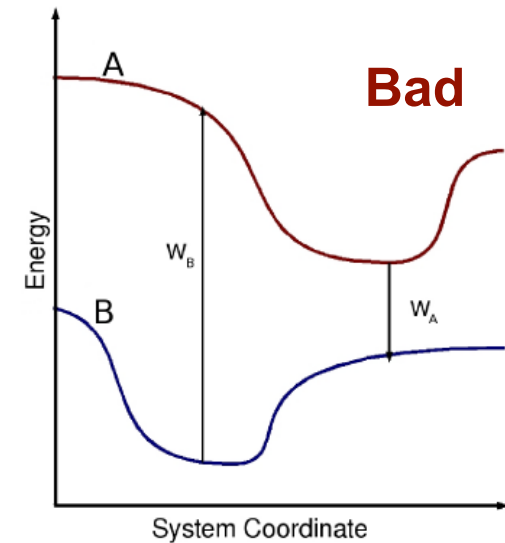
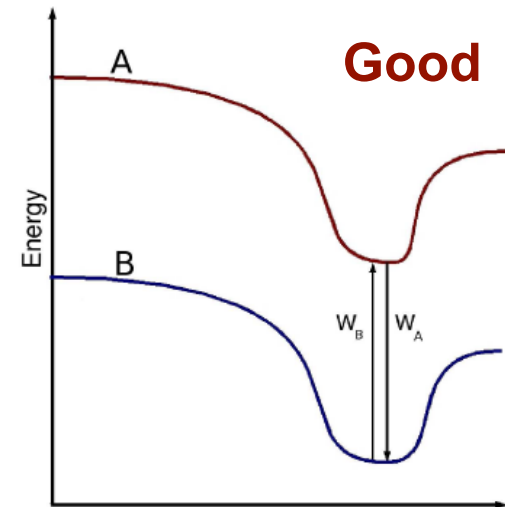
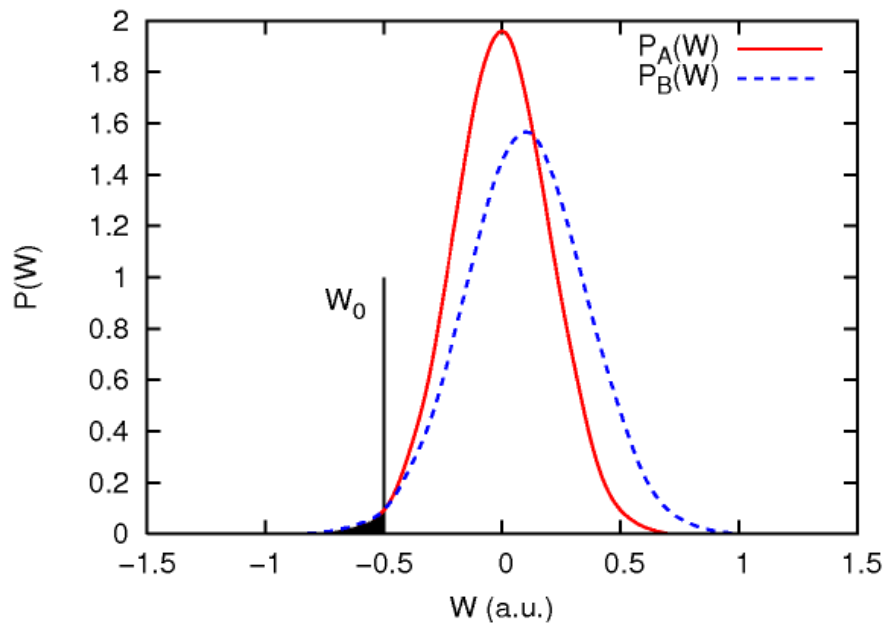
~100 Steps for Alloy

# Convergence Properties

## Some Theoretical Considerations

**Lu and Kofke (2001)**

$$\frac{\exp(-\beta\Delta F_{true}) - \exp(-\beta\Delta F_{calc,B})}{\exp(-\beta\Delta F_{true})} = \int_{-\infty}^{W_0} P_A(W) dW$$



# Summary

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- Quantum molecular dynamics simulations of molten superalloys
  - Good accuracy demonstrated for molar volumes, diffusivities (and mixing enthalpies)
  - Results provide data to refine molar-volume parametrizations for higher length-scale models in materials design
- Towards first-principles calculations of alloy solid-liquid phase equilibria and thermodynamic properties
  - Free-energy perturbation approach as framework for coupling classical and first-principles simulations for calculation of free energies with DFT accuracy
  - Promising convergence demonstrated for classical model system
  - Approach requires reasonably accurate classical-potential models